Quantitative Methods for a New Configuration of Territorial Units in a Chilean Government Agency Tender Process

Electronic Companion

Appendix A: TU Criterion Weights in Local Search Heuristic

We describe the calculation of the TU weights on each criterion ($x_{Rac,j,r}$, $x_{Col,j,r}$, $x_{Sup,j,r}$ and $x_{Acc,j,r}$) used in the heuristic methodology. Note that the partition of the region into TUs is known at the time the calculations are made. In the example we present below, we assume that a region is partitioned into two TUs, A and B. TU A serves 20,000 meals, covers an area of 8,000 km^2, and contains 90 schools, 4 of which are difficult to access.

- **Number of meals**

  The attractiveness value of each TU on this criterion is the sum of the percentage values for each constituent district. Clearly, a TU is more attractive the more meals it serves, because of economies of scale. Table 11 shows an example of the calculation of this weight.

TU	Number of meals	Attractiveness (meals)
A	20,000	40.0%
B	30,000	60.0%
Total	50,000	

Table 11: The table shows a calculation of TU attractiveness based on the number-of-meals criterion.

- **Number of schools**

  Because the attractiveness of a TU based on this criterion is inversely proportional to its size, we cannot simply add up the percentage attractiveness values of its individual districts to obtain the corresponding weight. To circumvent this problem, we first determine the number of schools in each TU formed by the heuristic and then derive an index by dividing the school total for the region by this number. Thus, we arrive at the index value of 2.22 for TU A by dividing 200 by 90 (see Table 12). We then normalize these index values to 100 percent to obtain the attractiveness weight for each TU.

TU	Number of schools	Index	Attractiveness (schools)
A	90	2.22	55.0%
B	110	1.82	45.0%
Total	200		

Table 12: The table shows a calculation of TU attractiveness based on the number-of-schools criterion.

- **Area**

We obtain the value on this criterion by determining the number of square kilometers covered by each TU formed by the heuristic. Because the attractiveness of a TU is inversely proportional to its area, we again calculate an index value, in this case by dividing the region's total area by the TU area. In our example, the index for TU A is 2.5, derived by dividing 20,000 by 8,000 (see Table 13). The index values are then normalized to 100 percent to obtain each TU's attractiveness.

TU	Size of area (km²)	Index	Attractiveness (area)
A	8,000	2.50	60.0%
B	12,000	1.67	40.0%
Total	20,000		

Table 13: The table shows a calculation of TU attractiveness based on area criterion.

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- **Accessibility**

To obtain the weight of a TU on this criterion, we calculate an accessibility index for each TU by dividing the number of easy-to-access schools by the number of difficult-to-access ones. If no schools are in the latter category, we set the number at 0.5 to avoid dividing by 0. In our example, the accessibility index of 21.50 for TU A is obtained by dividing 86 by 4 (see Table 14), and normalizing the index to 100 percent to obtain the attractiveness for each TU. However, this criterion is very sensitive to the number of difficult-to-access schools because of their small absolute number; one such school may unduly influence the result. Numerical "tricks" can be used to circumvent this problem. We derive a "softened" final weight in which 50 percent of its value is given by the original result, as just described, and the other 50 percent is the fraction of the total TUs in the region represented by a single TU (50 percent in the case of two TUs). Thus, in our example, the softened attractiveness for TU A is calculated as 0.5 * 65.7 + 0.5 * 50 = 57.9 (see Table 14).

TU	N° of easy-access schools	N° of difficult-access schools	Accessibility index	Attractiveness (accessibility)	"Softened" attractiveness
A	86	4	21.50	65.7%	57.9%
B	101	9	11.22	34.3%	42.1%
Total	187	13			

Table 14: The table shows a calculation of TU attractiveness based on accessibility criterion.

To arrive at the relative importance or attractiveness of each TU (see Table 15), we multiply the 2x4 matrix formed by the preference vectors for each TU on each criterion by the 4x1 criteria preference vector (see Table 5, main text), thus obtaining the 2x1 matrix with the final weights for each TU.

Feasible TUs formed by algorithm	Attractiveness (meals)	Attractiveness (schools)	Attractiveness (area)	Attractiveness (accessibility)
A	40.0%	55.0%	60.0%	57.9%
B	60.0%	45.0%	40.0%	42.1%

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Meals	38.1%
Schools	34.2%
Area	16.9%
Accessibility	10.8%

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Feasible TUs formed by algorithm	Attractiveness of TU
A	50.4%
B	49.6%

Table 15: The table shows the final calculation of TU attractiveness.

Finally, in the heuristic methodology, we obtain the final TU weights for a given region with knowledge of the configuration of the entire region, thus allowing us to compare each TU with the rest of the region containing it. Note that changing any district to another TU changes the scores of all TUs in the region, not just those directly involved in the change (because of the effect of the inversely proportional criteria).

Appendix B: Cluster Weights on Each Criterion in the ILP Model

In this methodology, the calculation of the weights of each cluster on each criterion must be performed without knowledge of how the rest of the region will be configured. This means we cannot use the same procedure we used in Appendix A in which we calculated the attractiveness of each TU given the complete partition of the region. Hence, we must calibrate the values of each cluster on each criterion for the ILP model such that for a given cluster, they are similar to those used with the heuristic. We do this by taking the value for a given TU from the heuristic, which will remain fixed in the present calculations; we obtain the rest of the values as a direct proportion to this fixed one. Using the same example as in Appendix A, Cluster A and Cluster B (corresponding to TU A and TU B in the heuristic) will have the same attractiveness on each criterion, while the clusters in all other partitions will have values that are proportional. In this methodology, the cluster scores in a given partition of a region do not necessarily add up to 100 (except for the number of meals criterion, which as a directly proportional criterion will sum to that figure), but will come close. Based on numerical experiments and the final results of this study, we found that this calibration leads to realistic calculations of the final attractiveness value for each cluster used in the ILP methodology.

- **Number of meals**

  Table 16 shows the weights obtained for each cluster on this criterion, which were calibrated so that the absolute values for Cluster A (20,000) and Cluster B (30,000) would yield relative attractiveness values of 40 and 60, respectively, the same percentages derived in the heuristic methodology analysis. The values for the rest of the clusters would be calculated so that the numbers of meals in each case maintain the same proportions. Clusters C and D, E and F, and G and H are examples of other partitions of the region.

Feasible clusters formed by algorithm	Number of meals	Calibrated attractiveness (meals)
A	20,000	40.0
B	30,000	60.0
C	32,500	65.0
D	17,500	35.0
E	25,000	50.0
F	25,000	50.0
G	35,000	70.0
H	15,000	30.0

Table 16: The table shows a calculation of cluster attractiveness based on the number-of-meals criterion.

- **Number of schools**

Table 17 shows the weights obtained for each cluster on this criterion, which were calibrated so that the relative attractiveness value of Cluster A would be 55, the same percentage derived in the heuristic methodology analysis. The values for the remaining clusters in the example are calculated to maintain the proportions reflected in the indices derived by dividing the number of schools in the region by the number in each cluster. For example, we obtain the attractiveness value of 49.5 by the calculation 55*2/2.22. Recall that clusters C and D, E and F, and G and H also form partitions of the region, as with Cluster A and Cluster B; however, the sums of the attractiveness values of the clusters of a single partition do not necessarily equal 100.

Feasible clusters formed by algorithm	Number of schools	Index	Calibrated attractiveness (schools)
A	90	2.22	55.0
B	110	1.82	45.0
C	100	2.00	49.5
D	100	2.00	49.5
E	80	2.50	61.9
F	120	1.67	41.3
G	110	1.82	45.0
H	90	2.22	55.0

Table 17: The table shows a calculation of cluster attractiveness based on the number-of-schools criterion.

- **Area**

Table 18 shows the weights obtained for each cluster on this criterion, which we calibrated so that the relative attractiveness value of Cluster A would be 60, the same percentage that the heuristic generated. The values for the remaining clusters are calculated to maintain the proportions reflected in the indices derived by dividing the number of square kilometers in the entire region by the number of square kilometers in the cluster. For example, the attractiveness value of 43.6 for cluster C is given by the calculation 60*1.82/2.50.

Feasible clusters formed by algorithm	Size of area (km²)	Index	Calibrated attractiveness (area)
A	8,000	2.50	60.0
B	12,000	1.67	40.0
C	11,000	1.82	43.6
D	9,000	2.22	53.3
E	10,000	2.00	48.0
F	10,000	2.00	48.0
G	12,000	1.67	40.0
H	8,000	2.50	60.0

Table 18: The table shows a calculation of cluster attractiveness based on the area criterion.

- **Accessibility**

Table 19 shows the values obtained for each cluster on this criterion, calibrated and softened to make the final attractiveness factor for TU A 57.9 (the percentage calculated in the heuristic case). To arrive at these values, we begin by calculating each TU's attractiveness on the accessibility criterion based on the value of 65.7 for TU A. The rest of the values for obtaining the calibrated attractiveness on accessibility are calculated maintaining the proportions given by the accessibility index, which is the ratio of easy-to-access to difficult-to-access schools of each cluster. The attractiveness value of 30.9 for cluster C, for example, is given by 65.7*10.11/21.50. To then obtain the final (softened) attractiveness values, recall that the value just calculated accounts for 50 percent, while the other 50 percent should be provided by the fraction each cluster represents in the total number of clusters in a given configuration of a region. However, because with this methodology we do not know the complete configuration *a priori*, we obtain this fraction based on cells rather than TUs. Thus, we divide the number of cells in the cluster by the total number of cells within the region.

Feasible clusters formed by algorithm	N° of easy-access schools	N° of difficult-access schools	Accessibility index	Calibrated attractiveness (accessibility)	Calibrated and softened attractiveness (accessibility)
A	86	4	21.50	65.7	57.9
B	101	9	11.22	34.3	42.1
C	91	9	10.11	30.9	40.4
D	96	4	24.00	73.3	61.7
E	76	4	19.00	58.1	54.0
F	113	7	16.14	49.3	49.7
G	102	8	12.75	39.0	44.5
H	86	4	21.50	65.7	57.9

Table 19: The table shows a calculation of cluster attractiveness based on the accessibility criterion.

As with the heuristic, using this methodology, we derive the final attractiveness of a cluster by multiplying the matrix of preference vectors for each cluster on each criterion (8x4) by the criteria preference vector (4x1), thus obtaining the attractiveness values for each cluster (see Table 20).

Feasible clusters formed by algorithm	Attractiveness (meals)	Attractiveness (schools)	Attractiveness (surface)	Attractiveness (accessibility)
A	40.0	55.0	60.0	57.9
B	60.0	45.0	40.0	42.1
C	65.0	49.5	43.6	40.4
D	35.0	49.5	53.3	61.7
E	50.0	61.9	48.0	54.0
F	50.0	41.3	48.0	49.7
G	70.0	45.0	40.0	44.5
H	30.0	55.0	60.0	57.9

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Meals	38.1%
Schools	34.2%
Area	16.9%
Accessibility	10.8%

Feasible clusters formed by algorithm	Attractiveness of TU
A	50.4
B	49.6
C	53.4
D	45.9
E	54.2
F	46.6
G	53.6
H	46.6

Table 20: The table shows the final calculation of cluster attractiveness.

Finally, we calculate the attractiveness of each cluster without knowing what occurs in the rest of the region. Therefore, we must calibrate the values to obtain scores that maintain proportionality between the clusters.