Inaugural INFORMS TSL Workshop, *June 27, 2011, Asilomar, CA*

More Equitable Congestion-Mitigation Polices for Multimodal Transportation Networks*

Yafeng Yin
Department of Civil and Coastal Engineering
University of Florida

**Joint work with Di Wu, Toi Lawphongpanich and Hai Yang*

Background

- Congestion-Mitigation Polices
  - Congestion pricing
  - Tradable credit schemes (Verhoef et al., 1997; Yang and Wang, 2011)
    - Government distributes credits to travelers
    - Credits are charged for using transportation services
    - Credits can be traded among all the travelers
- Equity is critical for both policies

Background (Cont’d)

- Despite the fact that successful implementations exist, congestion pricing remains very tough to sell
- Much of the public opposition centers on the perceived inequalities
  - Congestion pricing harms the poor who may have to pay more due to their inflexible schedules or switch to less desirable routes, departure times or transportation modes.

Background (Cont'd)

- **We attempt to design more equitable pricing/credit schemes to alleviate congestion or improve social benefit in multimodal urban transportation networks**
- **Existing pricing models are not able to capture the distributional effects of pricing schemes on different income groups and thus do not offer meaningful discussions on the income-based equity**

Objective

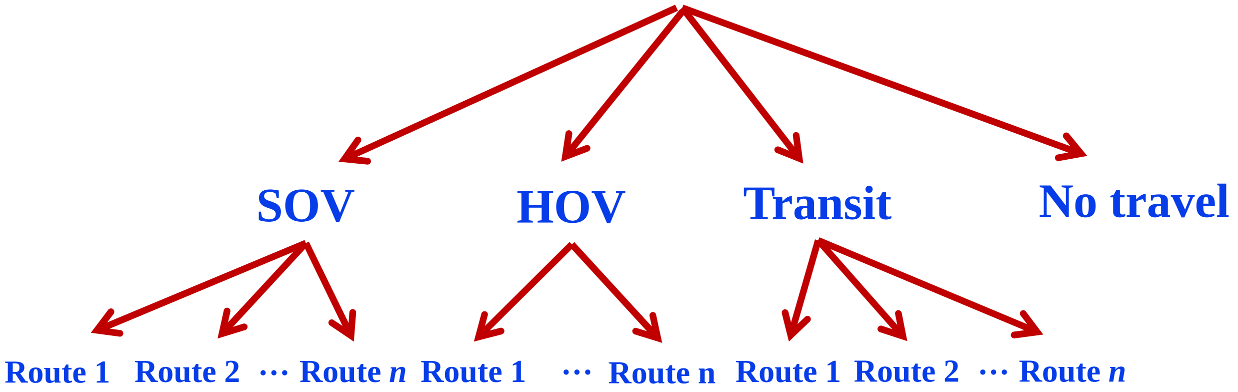
- **Design more equitable pricing/credit schemes by**
  - **Directly taking into account the effects of income on choices of trip generation, mode and route**
  - **Explicitly capturing the distributional impacts of pricing/credit schemes across different income and geographic groups**
- **A pricing/credit scheme is deemed to be more equitable if it leads to a more uniform distribution of wealth across population**

Basic Considerations

- **A general multimodal transportation network**
  - **Three types of facilities, i.e., transit services, high-occupancy/toll (HOT) and regular lanes**
- **Multiple user groups with different incomes and different preferences among four modes:**
  - **No travel, transit, drive alone (SOV), and car pool (HOV)**
  - **The number of travelers between each OD pair is fixed**

Choices of Mode and Route

- **Nested Logit model**



SOV **HOV** **Transit** **No travel**

Route 1 **Route 2** **⋯ Route *n*** **Route 1** **⋯ Route n** **Route 1** **Route 2** **⋯ Route *n***

$$P_k^{w,m,g} = \frac{\exp(\frac{v_k^{w,m,g}}{\theta^{w,m,g}})}{\sum_{j\in K^{w,m}} \exp(\frac{v_j^{w,m,g}}{\theta^{w,m,g}})} \cdot \frac{\exp(\bar{v}^{w,m,g})}{\sum_{m'\in M} \exp(\bar{v}^{w,m,g})}$$

$$\bar{v}^{w,m,g} = ln\left(\sum_{j\in K^{w,m}} \exp\left(\frac{v_j^{w,m,g}}{\theta^{w,m,g}}\right)\right)^{\theta^{w,m,g}}$$

Utility Function

- **Linear-in-income**

$$v = \beta_0 + \beta_1 T + \beta_2 (y_0 - \tau)$$

Travel Time

Income

Toll

  - **The above conventional specification with constant marginal utility of income may lead to an underestimate of the regressivity of a pricing scheme (e.g., Franklin, 2006; Bureau and Glachant, 2008)**

Nonlinear Utility Function

- **Generalized Leonief**

$$\boldsymbol{v} = \beta_0 + \beta_1 T + \beta_2 \sqrt{T} + \beta_3 y + \beta_4 \sqrt{y} + \beta_5 \sqrt{T}\sqrt{y}$$

- **Translog**

$$\begin{aligned}\boldsymbol{v} = \beta_0 + \beta_1 \ln T + \beta_2 \ln^2 T + \beta_3 \ln y + \beta_4 \ln^2 y \\ + \beta_5 \ln T \ln y\end{aligned}$$

Pricing: $y = y_0 - \tau$

Credit: $y = y_0 + p(q - \tau)$

Price

Credits distributed

Credits paid

Tolled User Equilibrium

- **VI Formulation**

$(f^*, d^*) \in \Phi$ is in user equilibrium if

$$\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}}\left(-v_k^{w,m,g} + \theta^{w,m,g}\ln f_k^{w,m,g}\right)\cdot\left(f_k^{w,m,g} - f_k^{w,m,g*}\right)$$

$$+\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}(1-\theta^{w,m,g})\ln d^{w,m,g}\cdot\left(d^{w,m,g} - d^{w,m,g*}\right) \geq 0,$$

$$\forall (f,d) \in \Phi$$

User Equilibrium under Credit Scheme

- **VI Formulation**

$(f^*, d^*, p^*) \in \Phi$ is in equilibrium if

$$\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}}\left(-v_k^{w,m,g} + \theta^{w,m}\ln f_k^{w,m,g*}\right)\cdot\left(f_k^{w,m,g} - f_k^{w,m,g*}\right)$$

$$+\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}(1-\theta^{w,m})\ln d^{w,m,g*}\left(d^{w,m,g} - d^{w,m,g*}\right)$$

$$+\left(Q - \sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}}\sum_{l\in L}\Delta_{l,k}\kappa_l^m f_k^{w,m,g*}\right)\cdot(p-p^*)$$

$$\geq 0,$$

$$\forall (f,d,p)\in\Phi$$

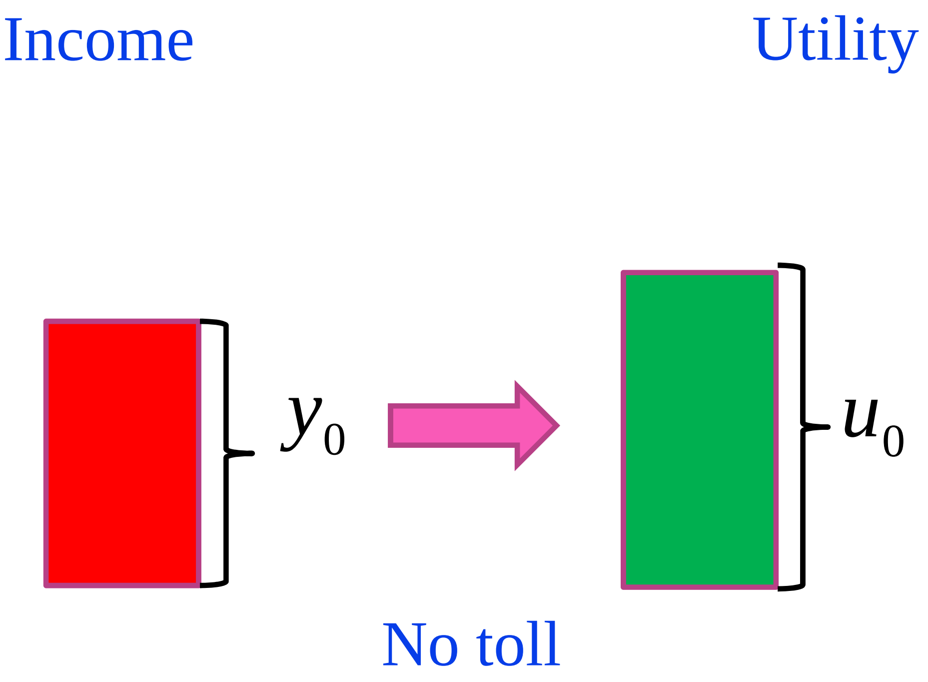
Welfare Measure

- **Equivalent Income**
  - **The income level that allows the individual to experience under the no-toll scenario the same level of utility as the original income does under the tolling scenario**

$$e^{w,g}(\tau) = arg\{z: u^{w,g}(z,0) = u^{w,g}(y^g,\tau)\}$$

Welfare Measure (Cont’d)

- **Equivalent Income**



Income

Utility

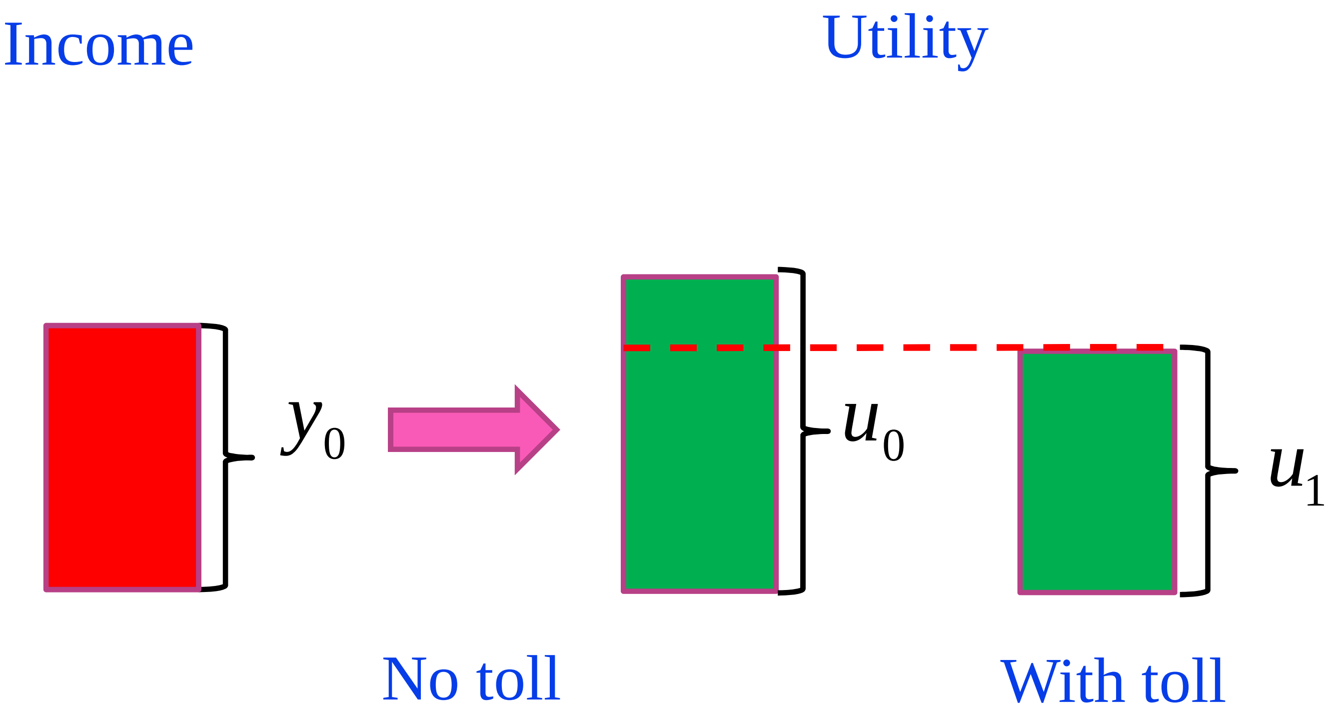
y_0

u_0

No toll

Welfare Measure (Cont'd)

- **Equivalent Income**



Income

Utility

y_0

u_0

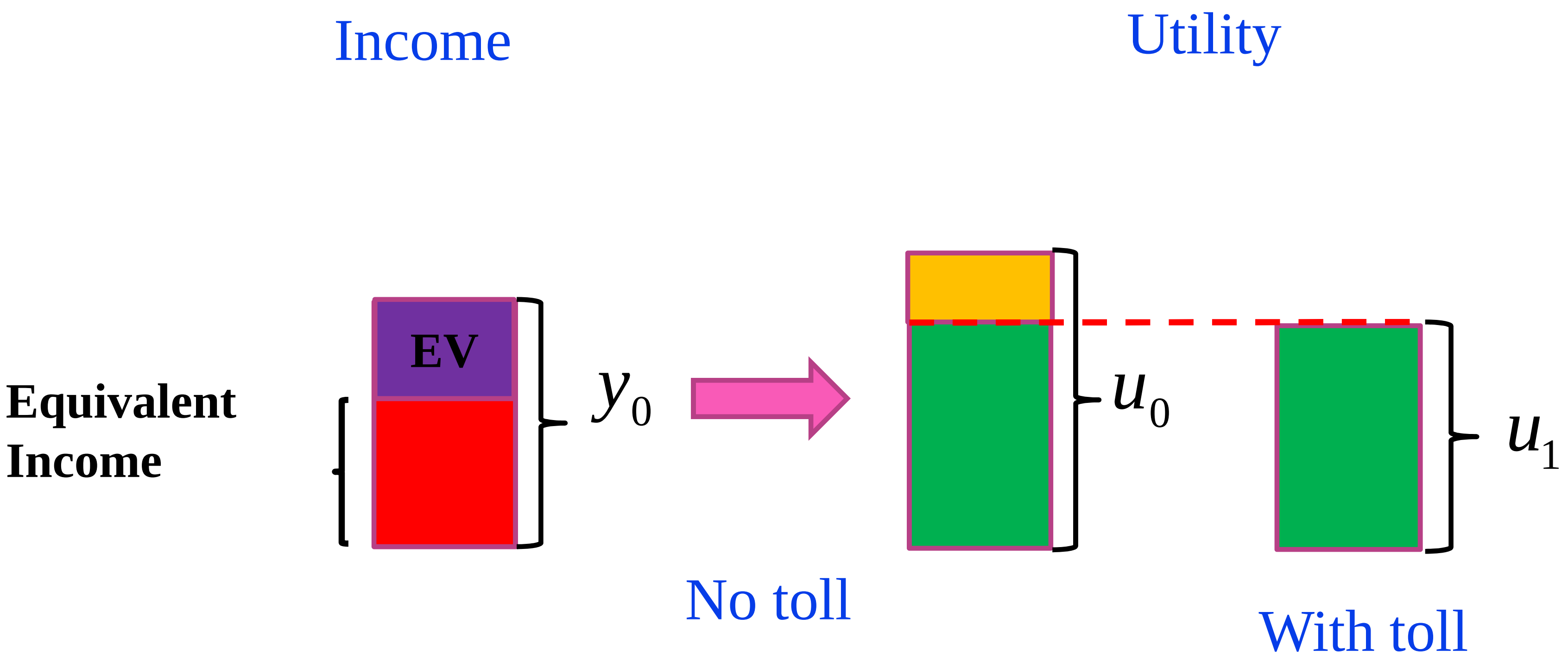
u_1

No toll

With toll

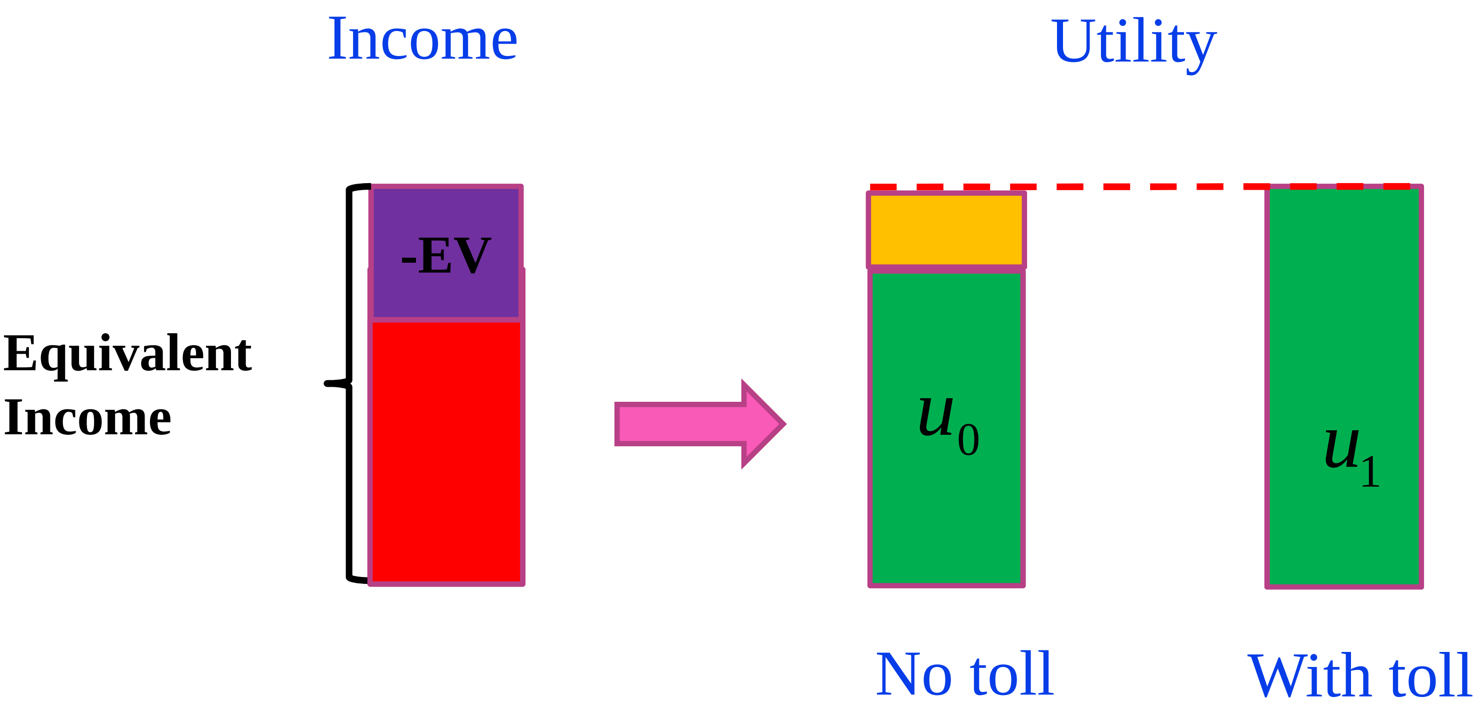
Welfare Measure (Cont'd)

- **Equivalent Income**



Welfare Measure (Cont’d)

- **Equivalent Income**



Welfare Measure (Cont’d)

- **Equivalent income is a measure of how wealthy a traveler feels under the pricing/credit scheme**
- **Due to the existence of the random error term in the utility function, the equivalent income is also random for each individual traveler**

$$e^{w,g}(\tau) = arg\{z: u^{w,g}(z,0) = u^{w,g}(y^{g},\tau)\}$$

- **Expected equivalent income**
  - **Dagsvik and Karlstrom (2005)**

Welfare Measure (Cont'd)

$$E\left(e^{w,g}(\tau)\right)$$

$$= \sum_{m^0 \in M} \sum_{k^0 \in K^{w,m^0}} \int_0^{p_{k^0}^{w,m^0,g}(\tau)} \frac{\left(\sum_{k \in K^{w,m^0}} \exp\left(\frac{h_k^{w,m^0,g}(z,\tau)}{\theta^{m^0}}\right)\right)^{\theta^{m^0}-1} \cdot \exp\left(\frac{v_{k^0}^{w,m^0,g}(y^g,0)}{\theta^{m^0}}\right)}{\sum_{m \in M}\left(\sum_{k \in K^{w,m}} \exp\left(\frac{h_k^{w,m,g}(z,\tau)}{\theta^m}\right)\right)^{\theta^m}} dz$$

$$v_{k^0}^{w,m^0,g}(y^g,\tau) = v_{k^0}^{w,m^0,g}(p_{k^0}^{w,m^0,g}(\tau),0)$$

$$h_k^{w,m^0,g}(z,\tau) = max\ (v_k^{w,m^0,g}(y^g,\tau), v_k^{w,m^0,g}(z,0))$$

Equity Measure

- **Gini coefficient**
  - **Calculated based on expected equivalent income with 0 being complete equality and 1 being complete inequality**

$$GN(\tau) = \frac{1}{2 \cdot \left(\sum_{g \in G} \sum_{w \in W} D^{w,g}\right)^2 \cdot \overline{E\left(e(\tau)\right)}} \cdot \sum_{g_1,g_2 \in G} \sum_{w_1,w_2 \in W} \left(D^{w_1,g_1} \cdot D^{w_2,g_2} \cdot \left|E\left(e^{w,g}(\tau)\right) - E\left(e^{w,g}(\tau)\right)\right|\right)$$

  - **A more equitable pricing/credit scheme will lead to a smaller value of the Gini coefficient**

Model Formulation

- **Objective**
  - **Efficiency: maximize the sum of the total expected equivalent income (user benefit) and the toll revenue (producer benefit)**
  - **Equity: minimize the Gini coefficient**

$$\max_{\tau,d,f} \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1-\alpha) \cdot \frac{GN(\tau)}{GN(0)}$$

Design Decisions

- **Congestion Pricing Scheme**
  - **Where to charge?**
  - **How much to charge?**

The feasible toll set Ψ can be defined as:

$$\tau_l^m \geq 0, \qquad \forall l \in L, m \in \{S, H\}$$

$$\tau_l^S = \tau_l^H, \qquad \forall l \in L_{RT}$$

$$\tau_l^R = 0, \qquad \forall l \in L$$

$$\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,g}} f_k^{w,m,g} \sum_{l \in L} \Delta_{k,l} \tau_l^m \geq 0$$

Design Decisions (Cont’d)

- **Tradable Credit Scheme**
  - **How to distribute the credits?**
  - **Where to charge the credits?**
  - **How many credits to charge?**

The feasible credit scheme set Ψ can be defined as:

$$\tau_l^m \geq 0, \qquad \forall m \in \{S, H, T\}, l \in L$$
$$\tau_l^R = 0, \qquad \forall l \in L$$
$$q^{w,g} \geq 0, \qquad \forall w \in W, g \in G$$
$$\sum_{w \in W} \sum_{g \in G} q^{w,g} D^{w,g} = Q$$

Design Model for Pricing

$$\max_{\tau,d,f} \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1-\alpha) \cdot \frac{GN(\tau)}{GN(0)}$$

$$s.t.$$

$$\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}} \left(-v_k^{w,m,g} + \theta^{w,m,g}\ln f_k^{w,m,g}\right) \cdot \left(f_k^{w,m,g} - f_k^{w,m,g*}\right)$$

$$+\sum_{g\in G}\sum_{w\in W}\sum_{m\in M} (1-\theta^{w,m,g})\ln d^{w,m,g} \cdot (d^{w,m,g} - d^{w,m,g*}) \geq 0,$$

$$\forall (f,d) \in \Phi$$

$$(f^*, d^*) \in \Phi$$

$$\tau \in \Psi$$

Design Model for Credit

$$\max_{d,f,p,q,\tau} \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1-\alpha) \cdot \frac{GN(\tau)}{GN(0)}$$

$s.t.$

$$\sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}} \left(-v_k^{w,m,g} + \theta^{w,m} \ln f_k^{w,m,g*}\right) \cdot \left(f_k^{w,m,g} - f_k^{w,m,g*}\right)$$

$$+ \sum_{g\in G}\sum_{w\in W}\sum_{m\in M} (1-\theta^{w,m}) \ln d^{w,m,g*} \left(d^{w,m,g} - d^{w,m,g*}\right)$$

$$+ \left(Q - \sum_{g\in G}\sum_{w\in W}\sum_{m\in M}\sum_{k\in K^{w,m}}\sum_{l\in L} \Delta_{l,k}\kappa_l^m f_k^{w,m,g*}\right) \cdot (p - p^*) \geq 0,$$

$$\forall (f, d, p) \in \Phi$$

$(f^*, d^*, p^*) \in \Phi$

$(q, \tau) \in \Psi$

Solution Algorithm

- Mathematical programs with equilibrium constraints (MPEC), a class of problems difficult to solve
- Compounding the difficulty is that the computation of the expected equivalent income involves numerical integrations
- Derivative-free algorithms
  - Compass search algorithm
  - SID-PSM algorithm (Custódio and Vicente, 2007; Custódio et al., 2010)

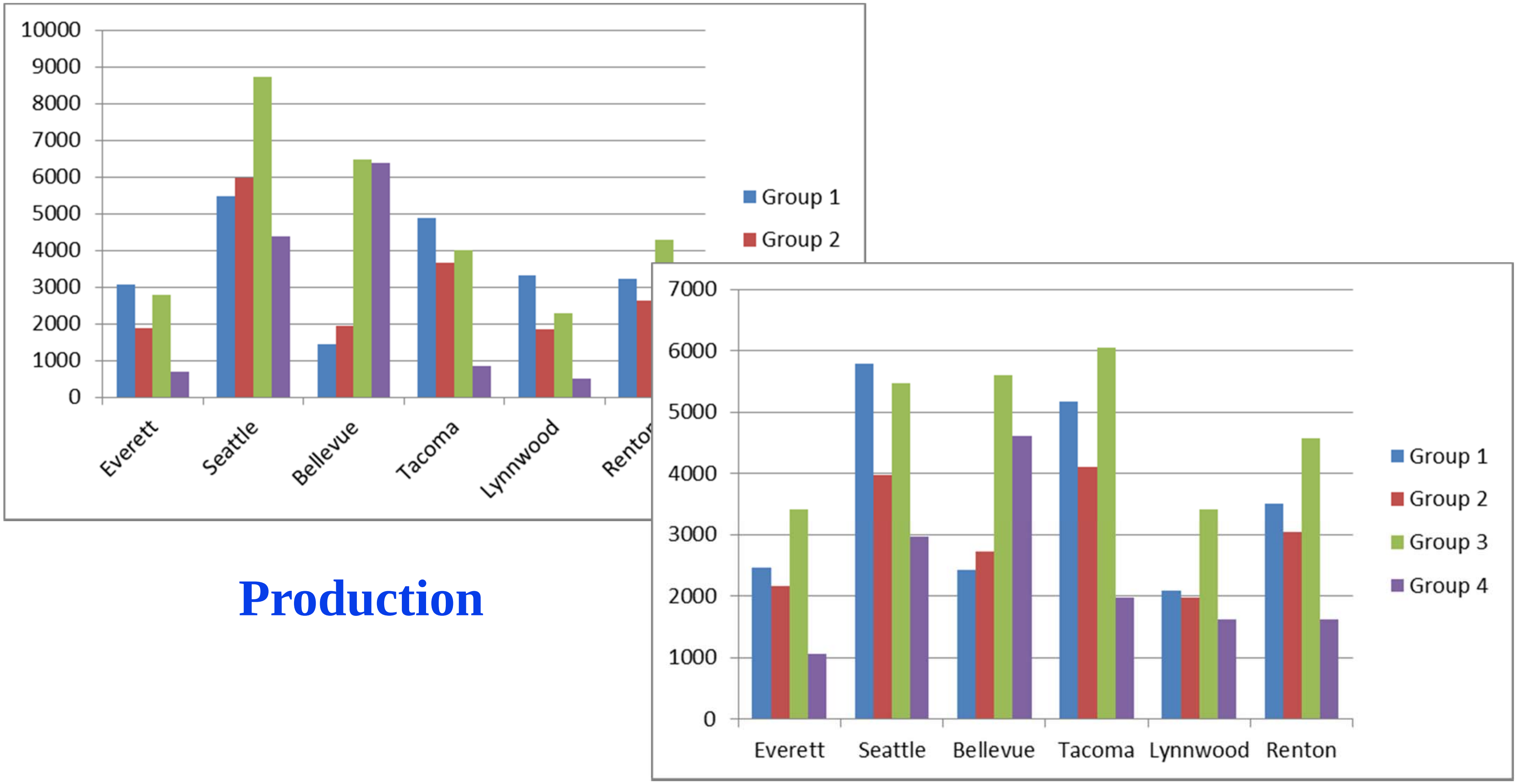
Numerical Example

- **Seattle Regional Freeway Network**
  - **Four income groups ($20,000; $40,000; $70,000; $120,000)**
  - **Translog utility function (Franklin, 2006)**

$$v_k^{w,m,g} = \beta_0^R \log \gamma y^g + \beta_1 \ln\left(y^g - C_k^{w,m}\right) + \beta_2 \ln T_k^{w,m} + \beta_3 \ln^2 T_k^{w,m}$$

UF UNIVERSITY of FLORIDA

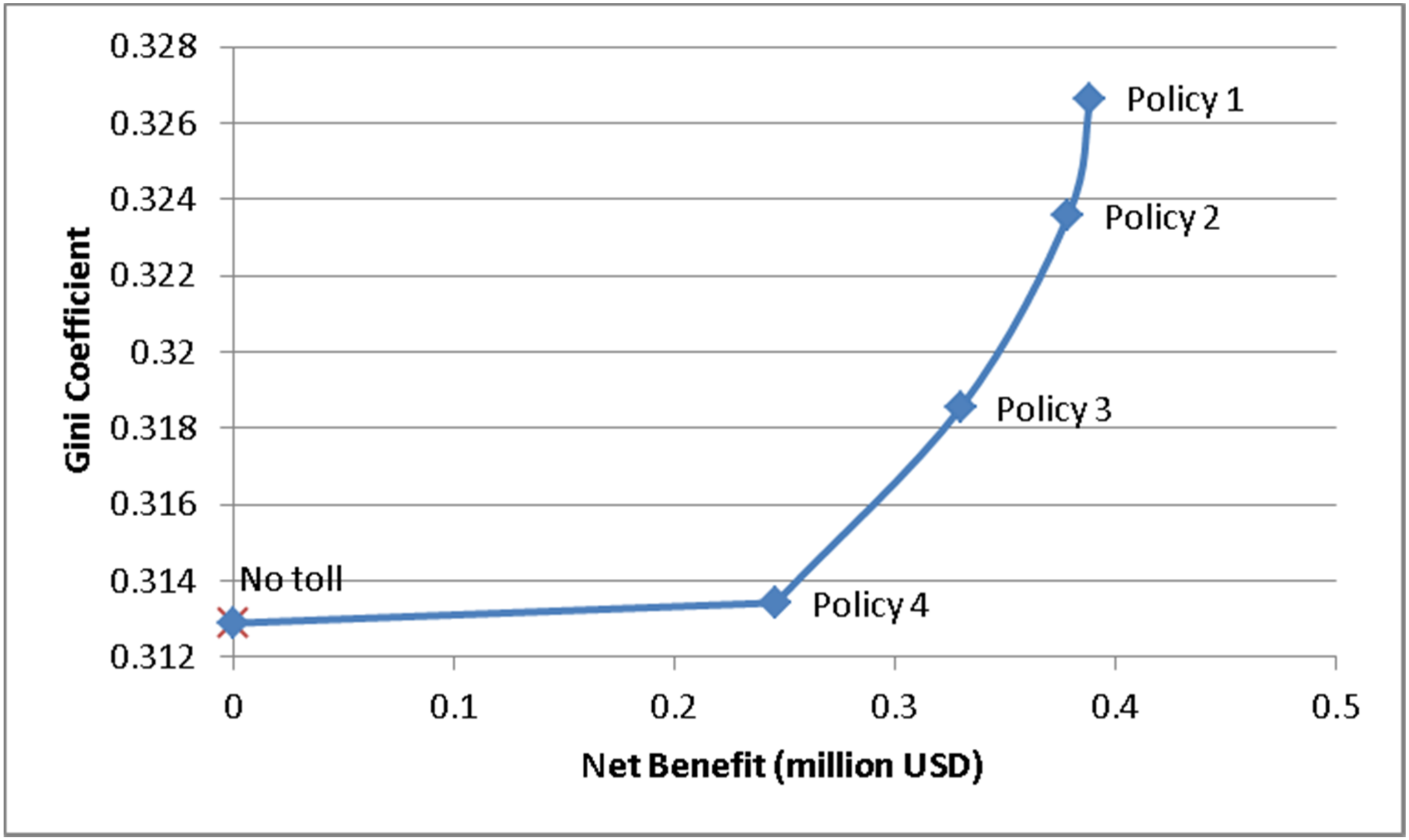
Travel Demand



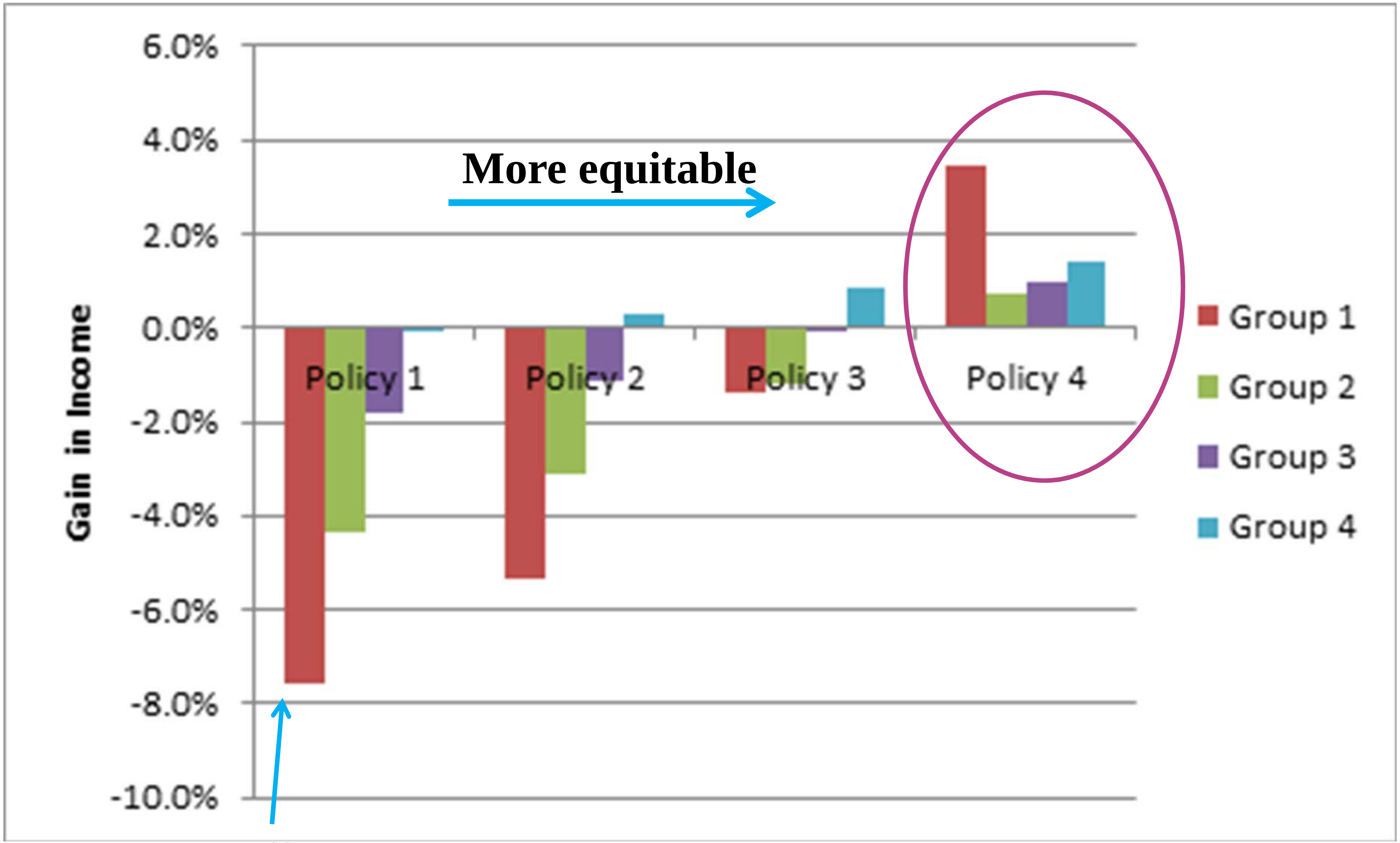
Production

Attraction

Optimal Pricing Schemes



Benefit Distribution



More equitable

Most efficient

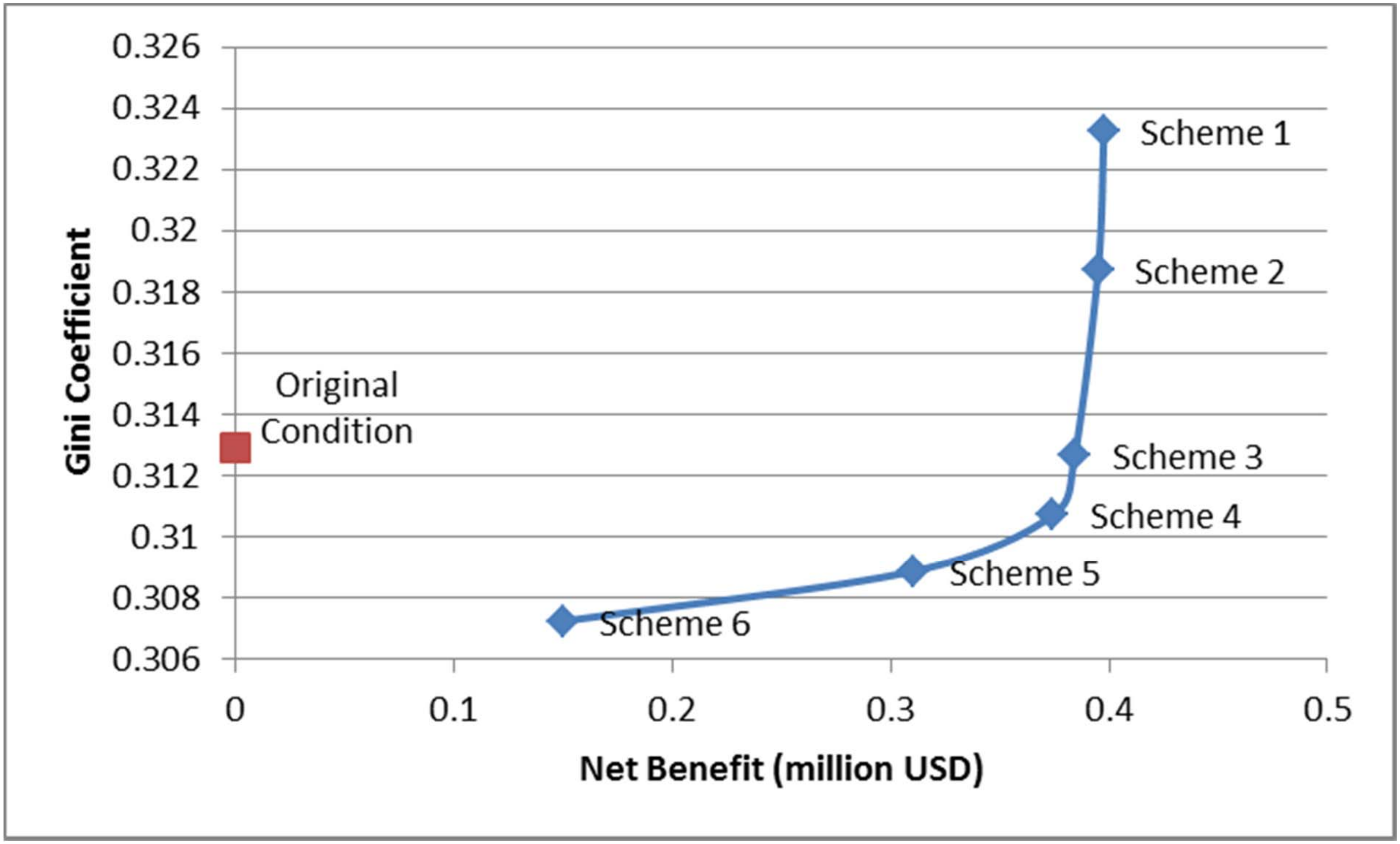
Optimal Pricing Schemes

More efficient

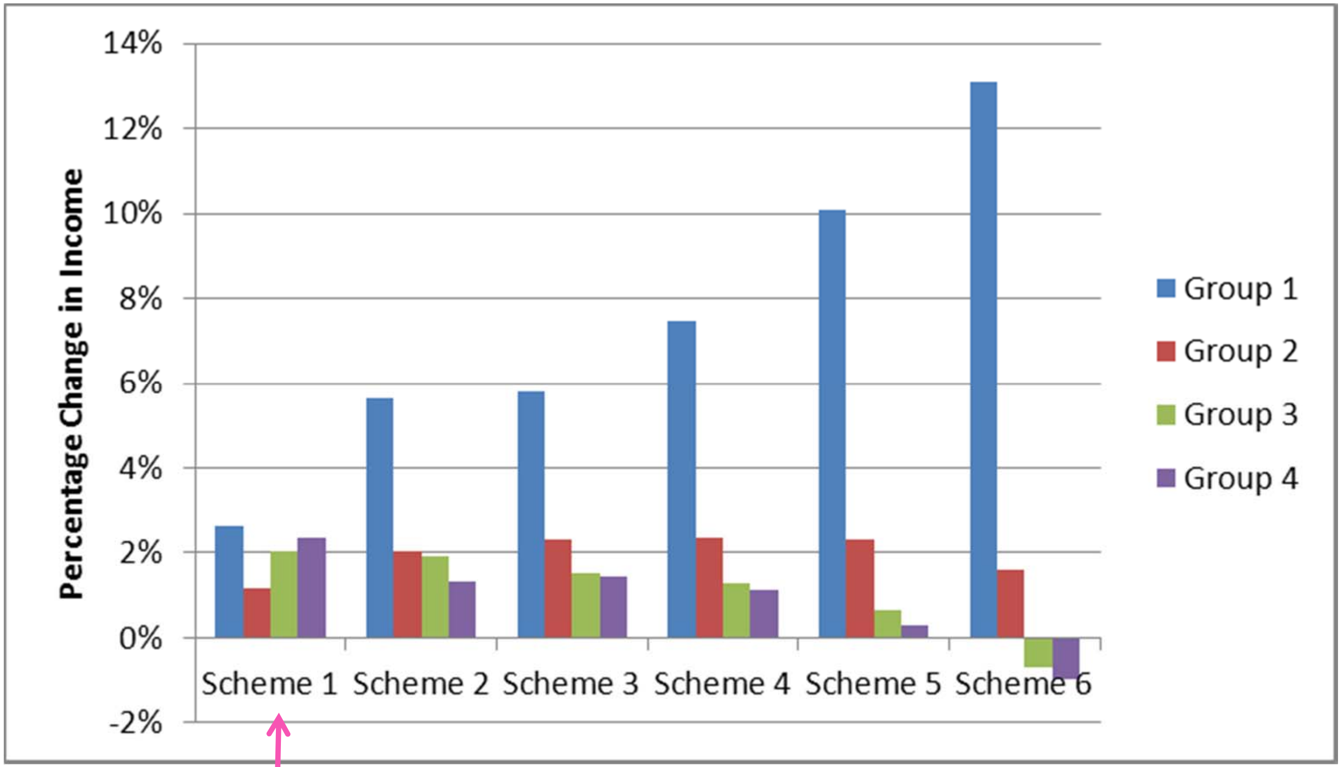
More equitable

	Policy 1		Policy 2		Policy 3		Policy 4	
Link	Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit
3	5.00	1.00	0.00	-1.00	0.00	-5.00	0.00	-15.00
4	5.00	1.00	1.00	0.00	0.00	-4.25	0.00	-15.00
5	10.25	0.00	8.25	-2.00	3.75	-10.00	2.25	-16.00
6	13.00	-1.00	13.00	-1.00	11.50	-4.00	12.50	-1.00
8	10.00	0.50	8.25	-2.50	3.25	-8.00	0.00	-15.50
9	13.00	-0.25	12.75	-2.25	10.75	-3.25	11.00	-0.25
12	10.00	0.00	10.00	0.00	7.75	-3.75	6.00	0.00
13	20.00	-2.00	20.00	-6.00	16.00	-12.00	12.00	-18.00
15	11.50	0.00	11.00	-1.75	9.00	-2.75	5.50	0.00
16	15.00	-2.00	15.00	-6.00	11.75	-10.00	7.00	-18.00
17	15.00	0.25	11.25	-5.50	6.75	-11.75	2.00	-20.00
20	8.00	-0.50	4.25	-4.50	0.75	-10.25	0.00	-16.50

Optimal Credit Schemes



Benefit Distribution

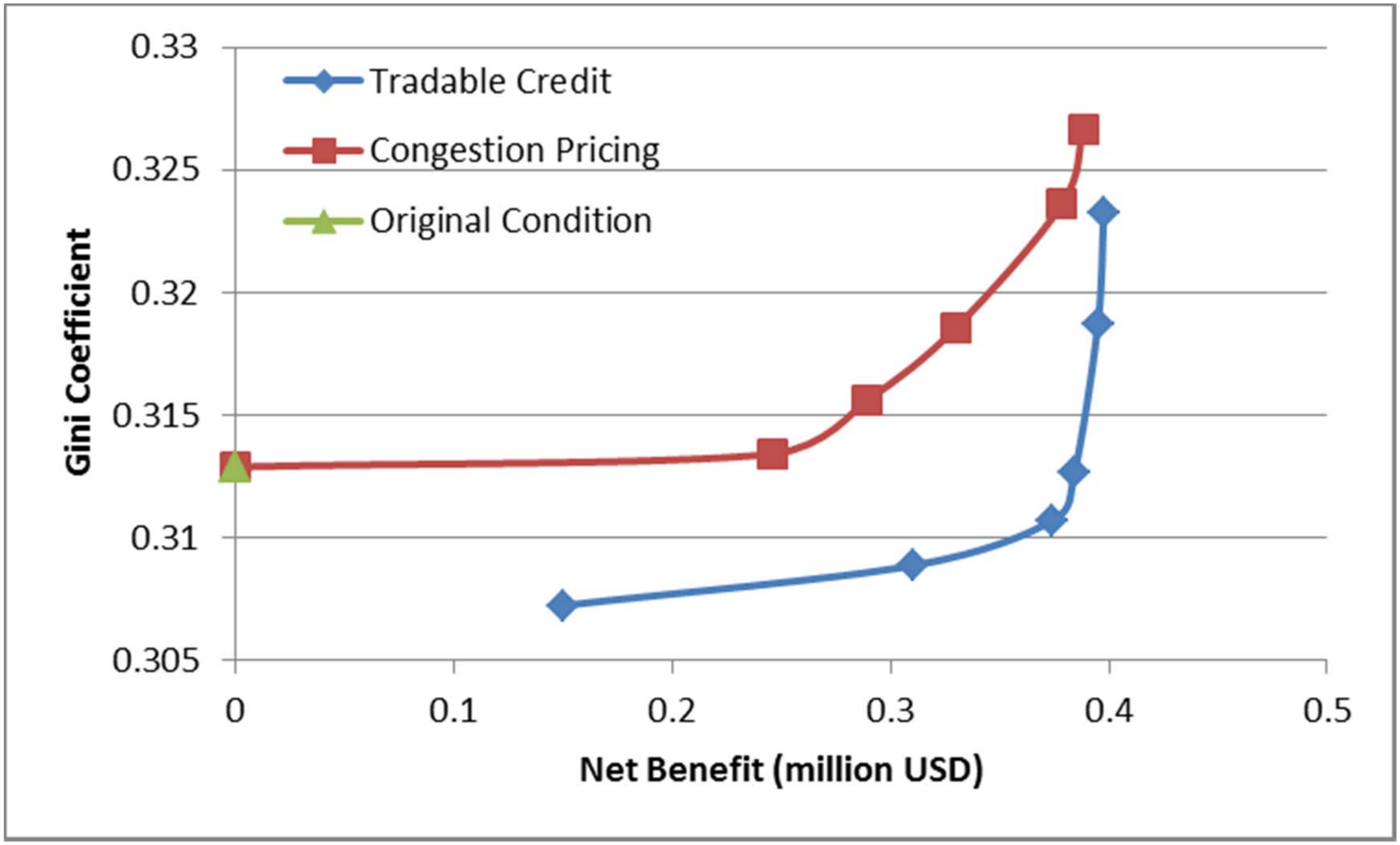


Most efficient

Credit Charging Schemes

		Scheme 1		Scheme 2		Scheme 3		Scheme 4		Scheme 5		Scheme 6	
Credit distributed	Everett	0.0		2.0		8.2		15.8		15.3		14.4	
	Seattle	0.0		0.0		13.1		9.2		6.8		3.4	
	Bellevue	13.1		0.0		0.0		0.0		0.0		0.0	
	Tacoma	0.0		0.8		7.0		11.6		15.6		21.8	
	Lynnwood	0.0		7.9		14.2		18.8		21.5		23.6	
	Renton	54.1		65.1		19.6		13.8		12.7		11.9	
		Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit
Credit charged	Link 3	11.3	2.6	10.8	1.9	5.5	0.1	5.8	0.3	4.1	0.6	6.6	1.5
	Link 4	8.0	2.1	8.4	1.8	8.8	0.2	14.9	1.3	19.9	2.1	19.9	14.5
	Link 5	11.8	0.6	12.1	0.6	14.0	1.6	12.3	0.3	11.5	0.8	14.6	4.1
	Link 6	12.9	0.0	13.3	0.0	18.6	0.0	19.9	1.7	19.9	2.5	19.9	14.9
	Link 8	13.1	0.8	13.4	1.0	15.3	1.6	15.2	0.2	19.9	2.2	19.9	14.4
	Link 9	15.8	0.5	15.7	0.2	15.7	0.2	16.8	1.1	20.0	3.5	19.9	15.8
	Link 12	9.7	0.0	10.2	0.0	16.1	0.0	16.6	2.0	19.9	3.1	19.9	15.4
	Link 13	20.0	0.0	20.0	0.0	20.0	0.0	20.0	0.2	20.0	0.9	20.0	5.0
	Link 15	14.3	0.6	13.7	0.2	13.4	0.5	14.9	1.2	18.9	3.6	19.9	15.9
	Link 16	16.9	0.0	15.5	0.0	12.3	0.0	12.5	0.0	16.4	0.0	17.9	10.9
	Link 17	17.3	0.7	17.5	0.6	14.4	0.9	13.9	0.6	13.4	1.0	16.3	6.4
	Link 20	6.3	0.0	5.7	0.0	4.4	0.0	4.5	0.0	7.9	0.0	20.0	11.3

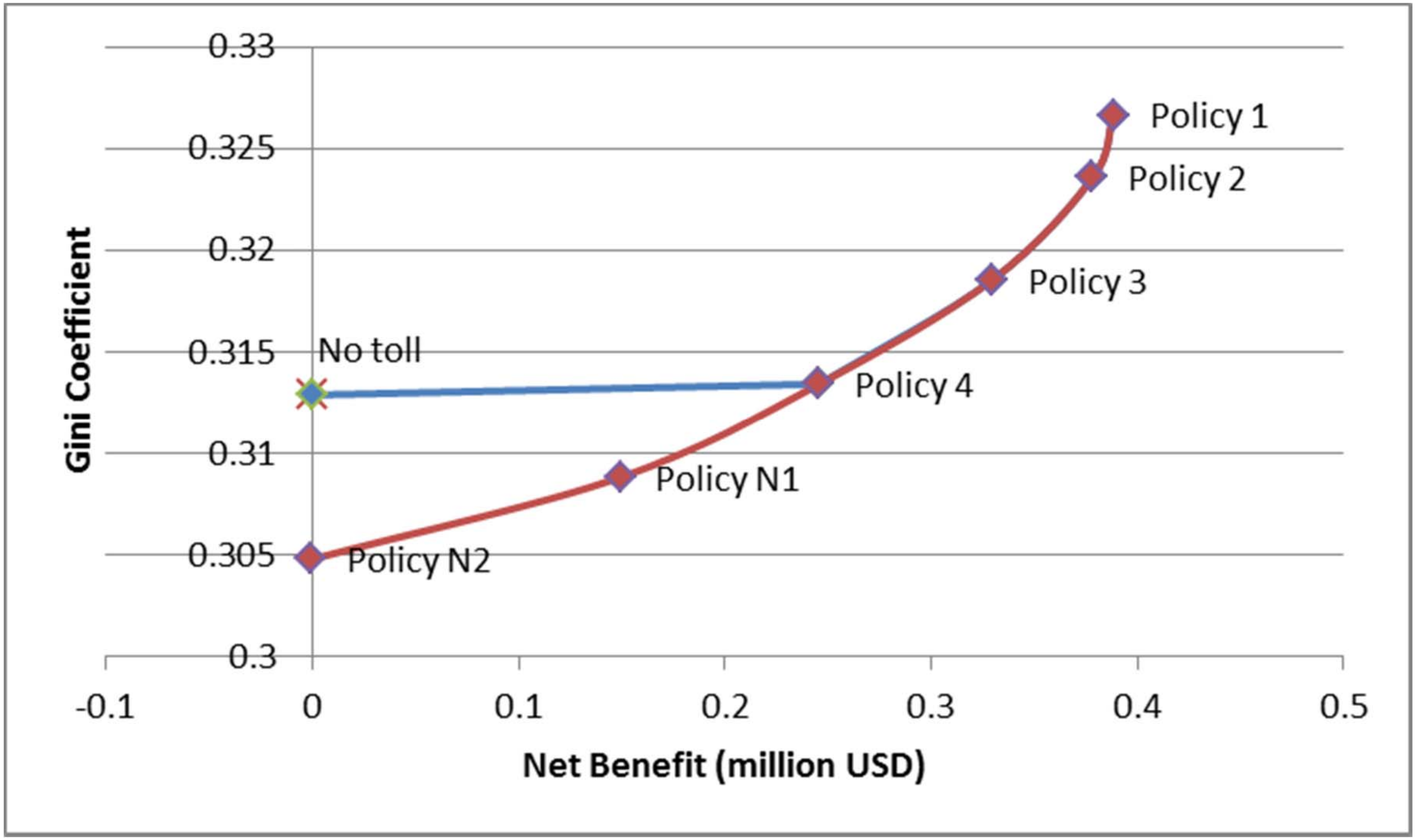
Comparison


0.33
0.325
0.32
0.315
0.31
0.305
Gini Coefficient
0
0.1
0.2
0.3
0.4
0.5
Net Benefit (million USD)
Tradable Credit
Congestion Pricing
Original Condition

Summary

- We developed a modeling framework that explicitly captures the distributional effects of pricing/credit schemes on different income and geographic groups
- The framework can be used to generate more equitable yet efficient pricing/credit schemes for multimodal transportation networks

Optimal Pricing Schemes (Cont'd)



Observations

- Low-income travelers suffer the most when efficiency is maximized
- When equity is given enough weight, low-income and high-income travelers both benefit more than mid-income travelers
  - Low-income travelers: transit subsidy
  - High-income travelers: reduction in travel time
- Better equity is achieved via heavier transit subsidy

Observations

- **Everybody is better off under the most efficient credit scheme.**
  - **Low-income travelers: selling extra credits**
  - **High-income travelers: reduction in travel time**
- **Better equity is achieved by increasing the number of credits charged at each link.**
  - **More credits charged ⟶ more demand for credits ⟶ higher credit price ⟶ more subsidies**