

# LOGIT-BASED PRICING

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# DETERMINISTIC CASE

$$\max_{t \in T} tx$$

$$\min_{x \in X} (c + t)x$$

# PATH FORMULATION

$$\max_{t,x} f(t,x) = \sum_{r \in \mathcal{R}} d_{q(r)} t_r x_r$$

$$t \in T$$

$$x \in \arg \min_y \{g^{DET}(t,y) = (c + t) \cdot y \mid y \in X\}$$

$$T = \{t \in \mathbb{R}^R, \exists t^A \in \mathbb{R}^{A_{\text{toll}}} : t_r = \sum_{a \in r} t_a^A\}$$

$$X = \{x \mid \sum_{r \in \mathcal{R}^q} x_r = 1, q \in \mathcal{Q}\}$$

# LOGIT MODEL

$$\max_t \sum_{q \in \mathcal{Q}} d_q \sum_{r \in \mathcal{R}^q} \text{logit}_r(u|\theta) \sum_{a \in r} t_a$$

$$\text{logit}_r(u|\theta) = \frac{\exp(-\theta u_r)}{\sum_{\ell \in \mathcal{R}^q} \exp(-\theta u_\ell)}$$

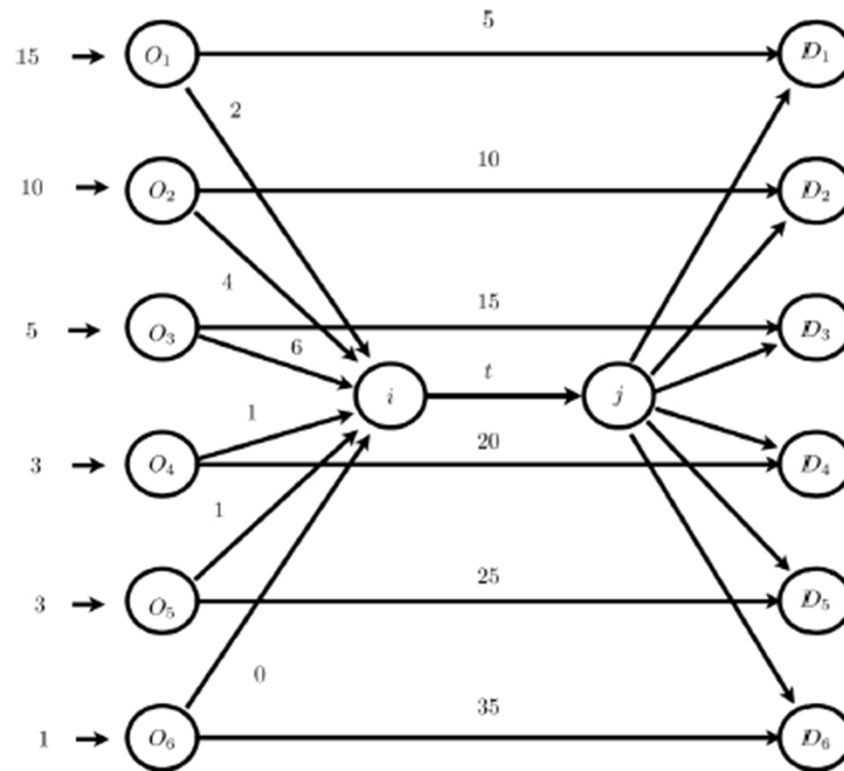
$$u_r = \sum_{a \in r} (c_a + t_a)$$

# BILEVEL FORMULATION

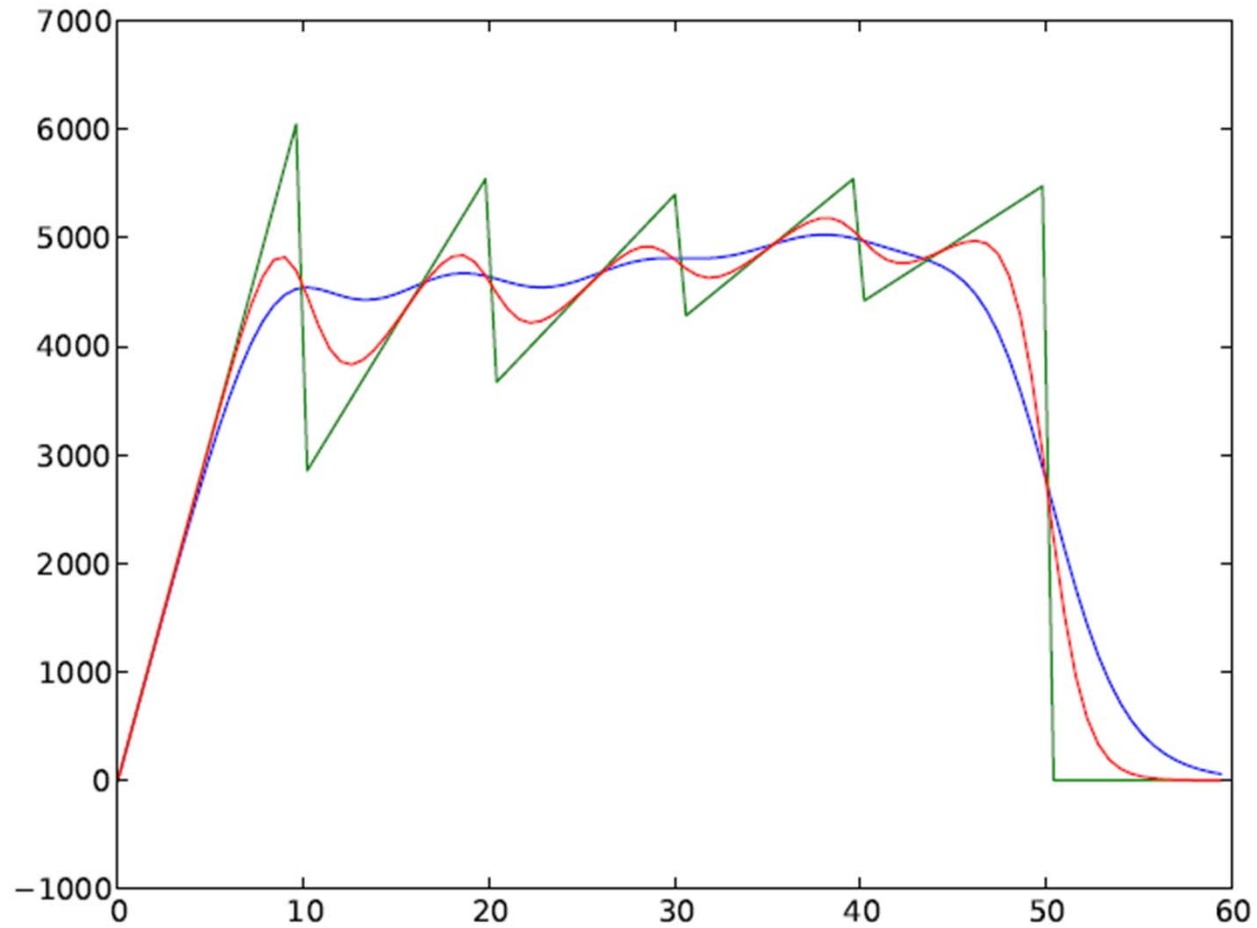
$$\max_{t \in T} f(t, x)$$

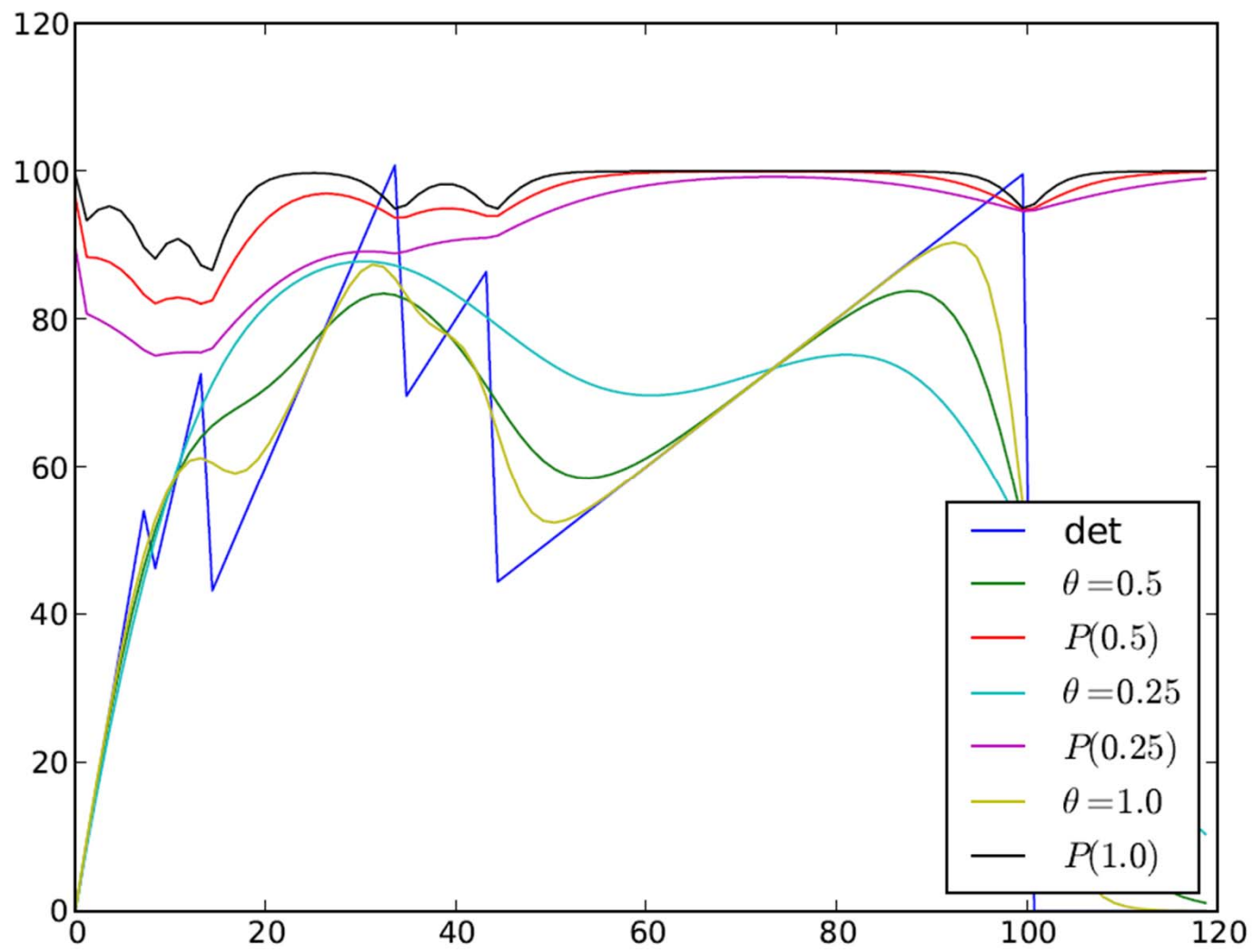
$$x \in \arg \min \{g(t, y) = (c + t) \cdot y + \frac{1}{\theta} y \cdot \log y \mid y \in X\}$$

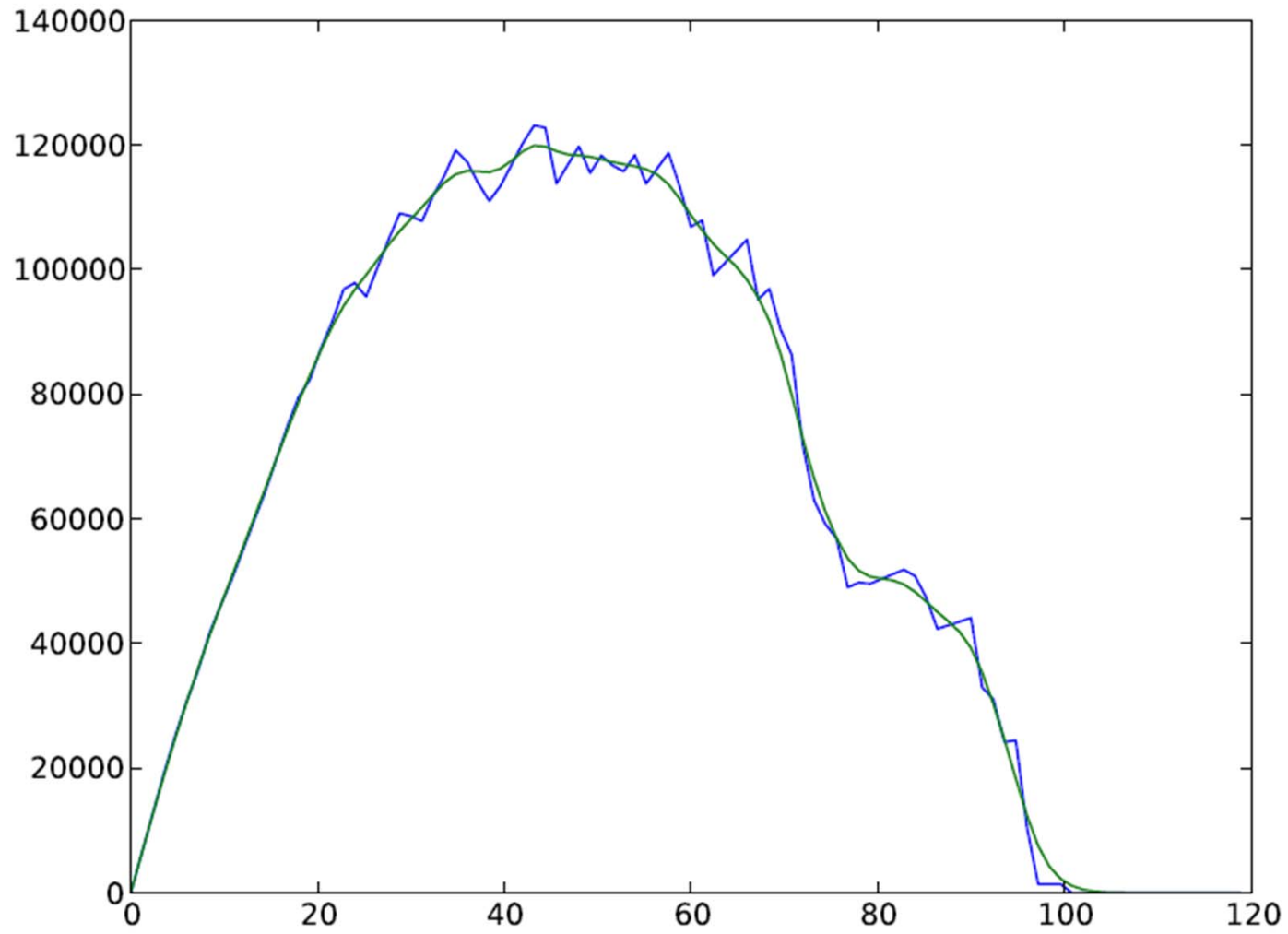
# OBJECTIVE FUNCTION



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50 O-D

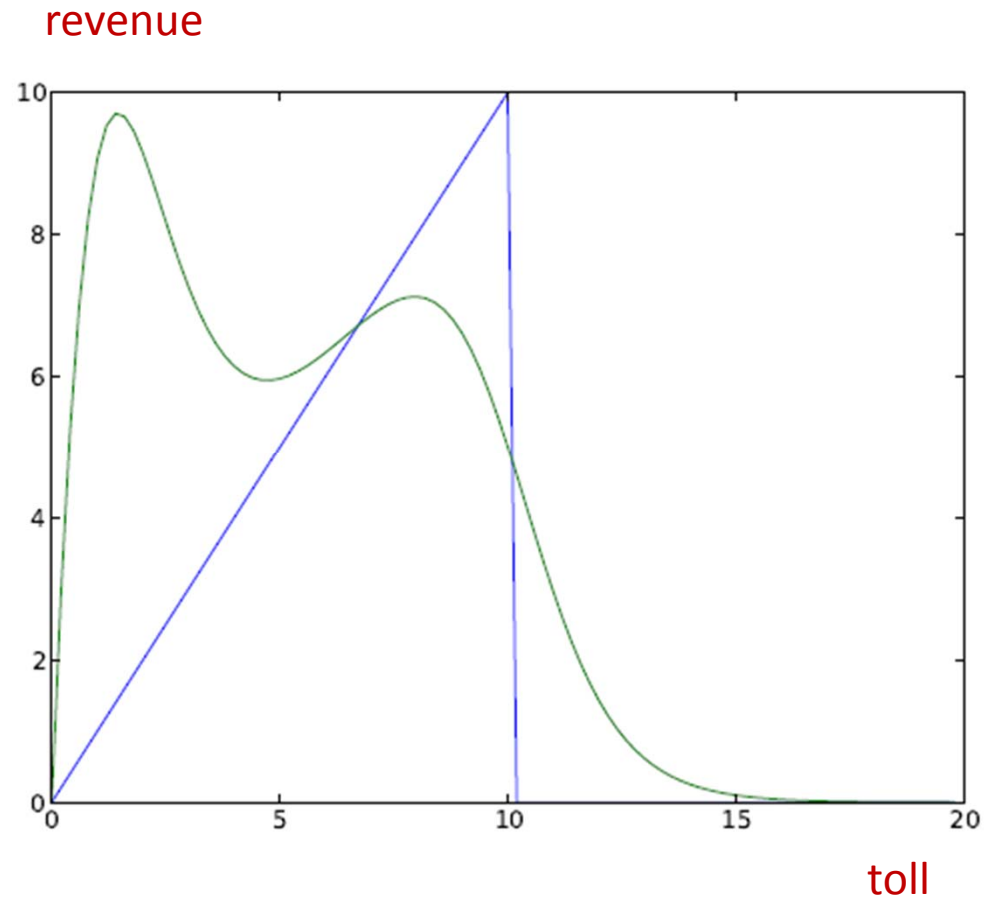
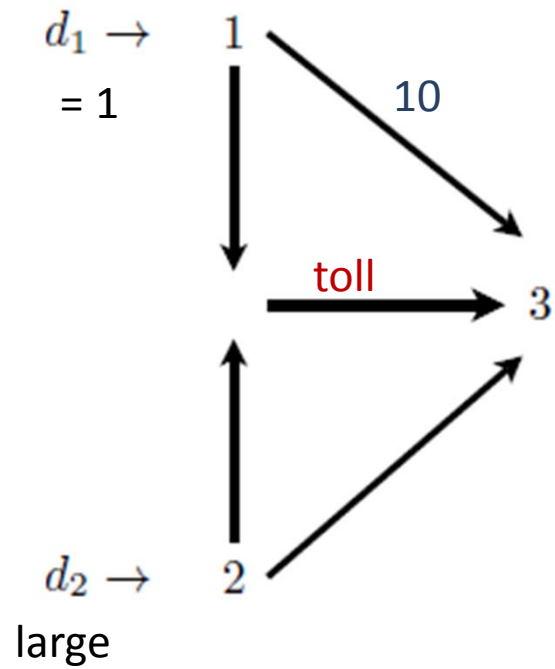
# ALGORITHMIC STRATEGIES

- Local search
- MIP approximations
- Trust region

# GRADIENT OF THE OBJECTIVE FUNCTION

$$\nabla f^{\text{logit}}(t) = \sum_{q \in \mathcal{Q}} d_q [p(t|\mathcal{R}^q) - \theta \text{COV}(t|\mathcal{R}^q) t']$$

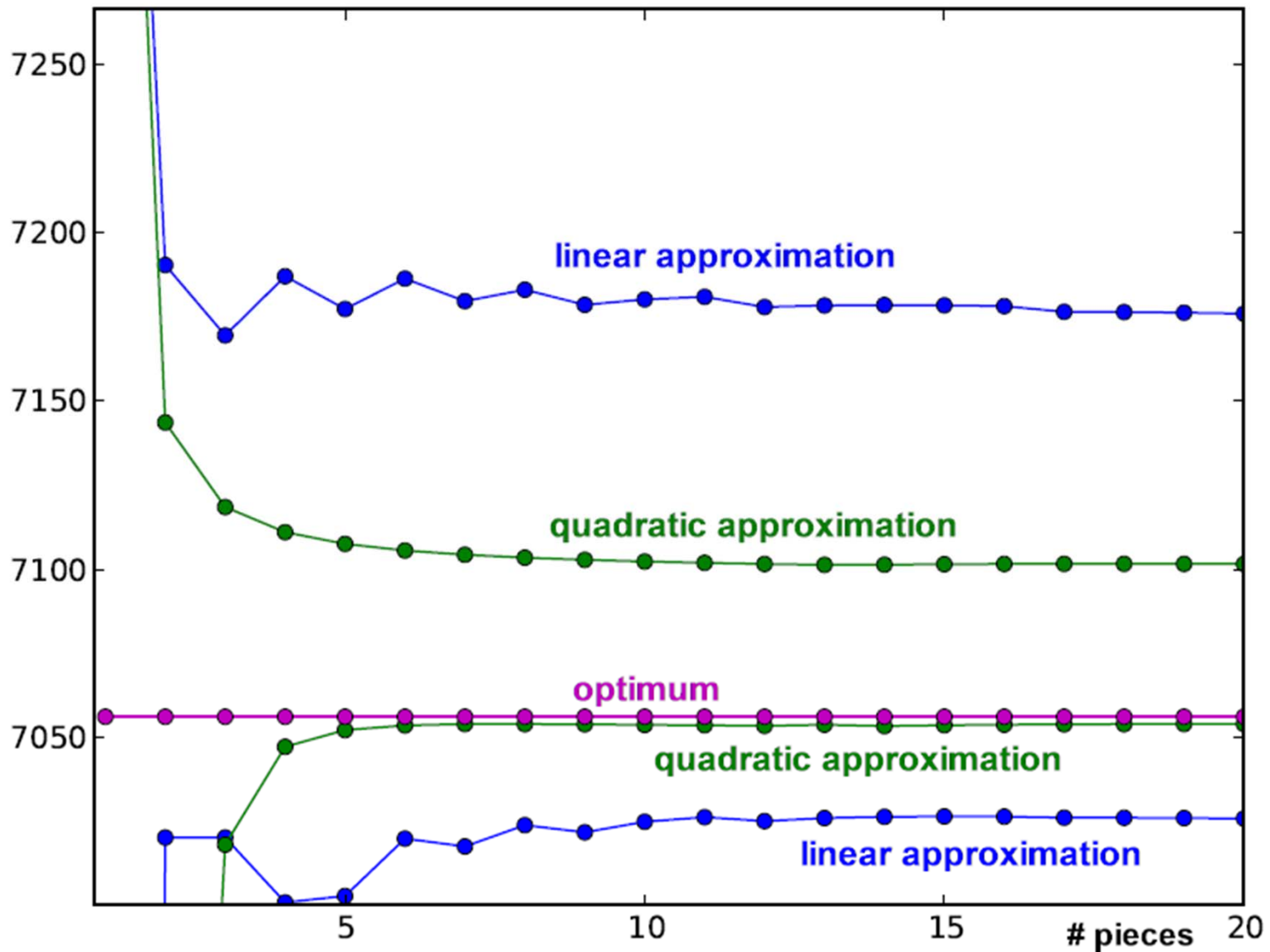
# DETERMINISTIC APPROXIMATION



# PIECEWISE APPROXIMATION OF THE LOGARITHMIC FUNCTION

- Theoretical result: optimality bound
- Linear or quadratic pieces

# PREDICTED VS. ACTUAL REVENUES

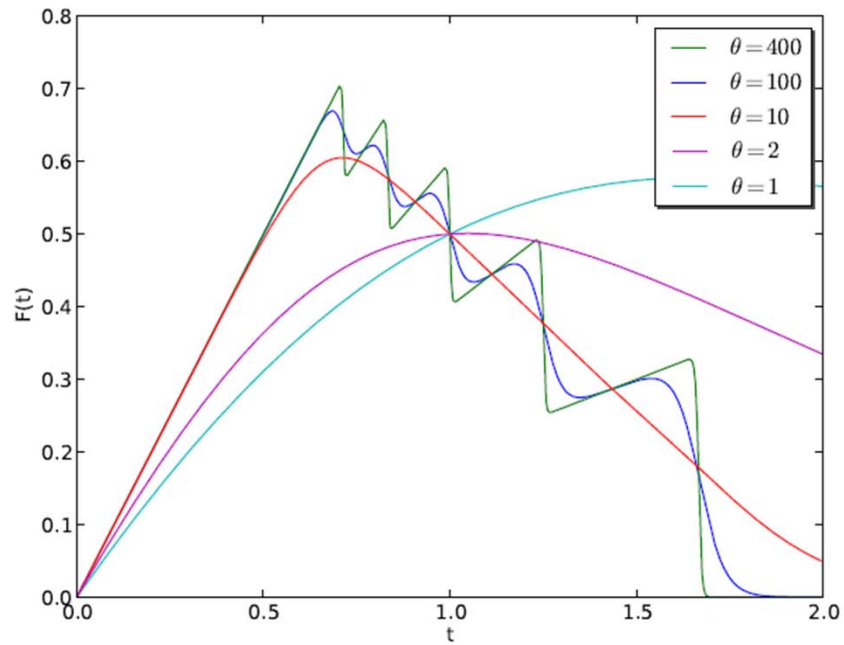


# SOME NUMERICAL RESULTS

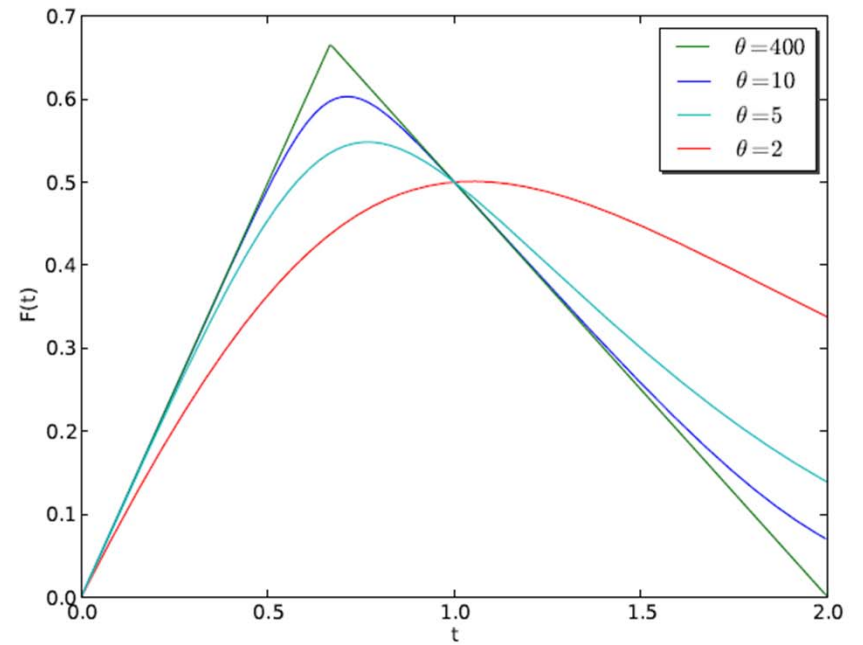
	DET	TR	PWL3	PWL3+TR	PWQ3	H50	TR-H7	DET+TR-H7
A-5-39	7056.02	5888.01	7056.02	7056.02	7056.02	4993.79	7056.02	7056.02
A-5-12	4802.51	4802.51	4802.51	4802.51	*	4802.51	4802.51	4802.51
A-5-48	13803.40	12789.20	13803.40	13803.40	13803.40	6434.33	13472.30	13803.40
A-5-46	6680.98	5803.71	6898.14	6898.14	6898.14	6898.14	5803.71	6680.98
A-5-28	6502.57	6502.57	6502.57	6502.57	6502.57	6487.53	6502.57	6502.57
A-5-79	11143.70	8912.30	11143.70	11143.70	11143.70	8438.40	8881.81	11113.20



# MIXED LOGIT (MULTICLASS)

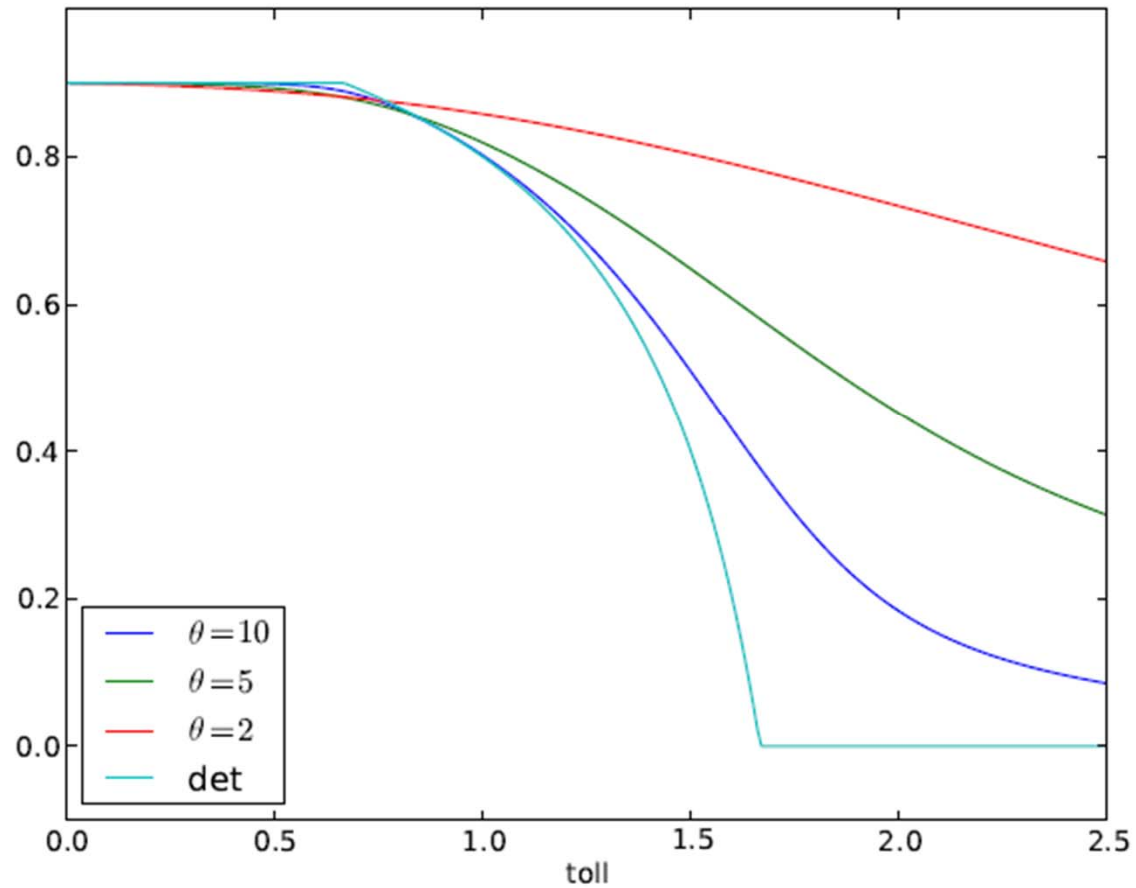


Discrete



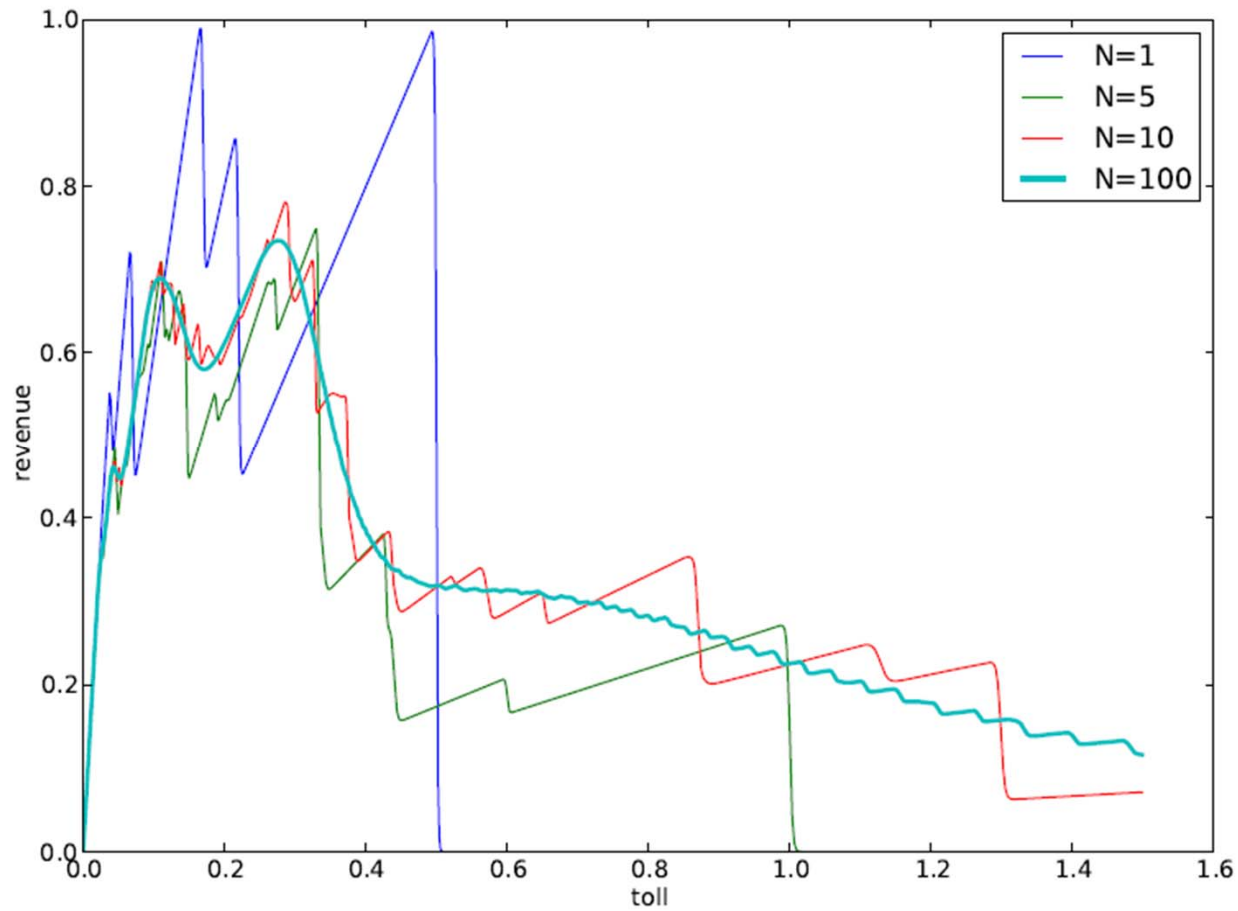
Continuous

# CLASS REVENUES

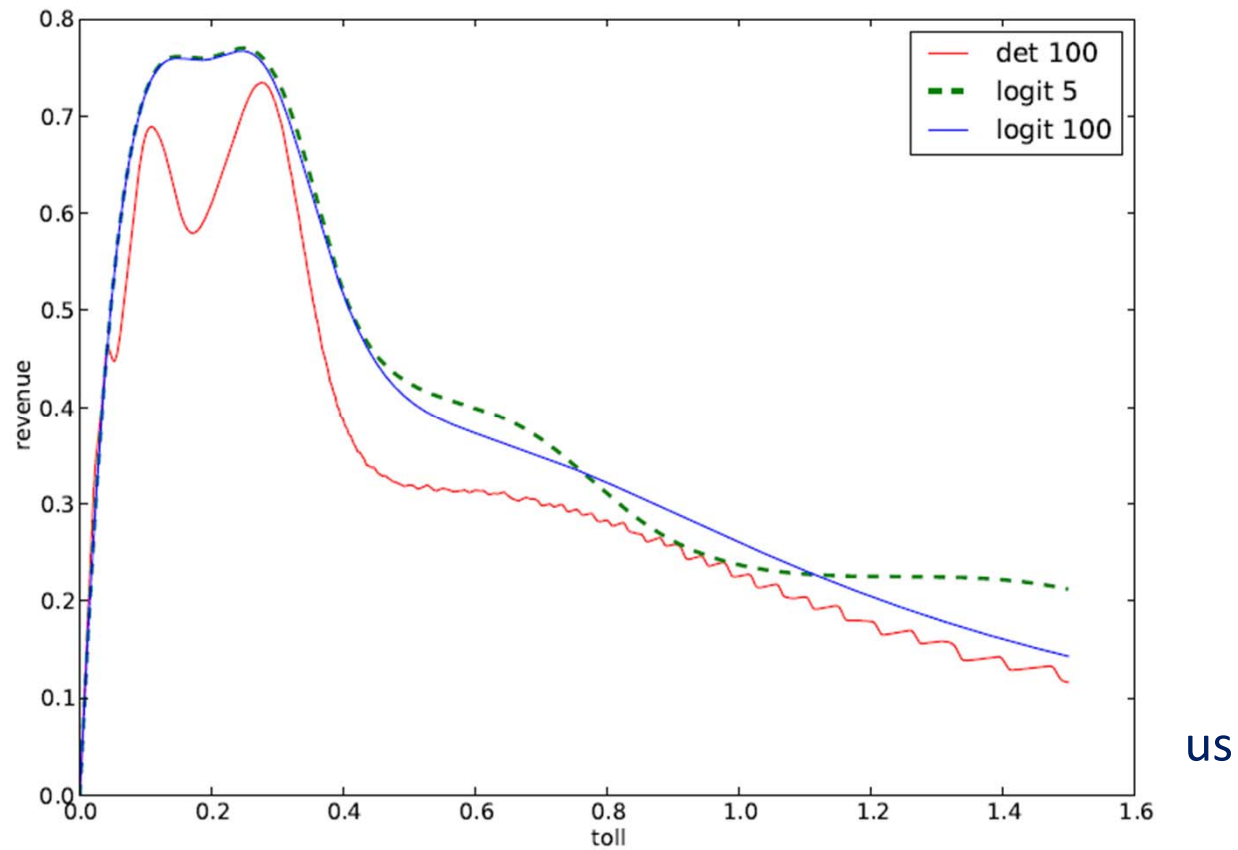


Proportion of revenue from  
price-sensitive customers

# INCREASING # CLASSES (DETERMINISTIC)



# MIXED LOGIT VS DETERMINISTIC



# EXTENSIONS

- Other discrete choice models of the GEV family (nested logit, etc.)
- Identifying the concavity zones
- Product line design problem
- Revenue management (capacities)
- Congestion

# SOME MORE NUMERICAL RESULTS

	DET	TR	PWL3	PWL3+TR	PWQ3	H50	TR-H7	DET+TR-H7
A-5-39	62.47	0.12	0.51	4.41	0.53	807.96	79.09	10.40
A-5-12	30.19	0.00	17.63	0.15	*	162.85	4.92	0.15
A-5-48	73.13	0.01	0.31	0.31	2.25	216.34	28.22	7.68
A-5-46	169.08	0.04	2.27	0.31	1.26	323.26	3.17	15.10
A-5-28	94.16	0.03	0.99	7.99	0.37	1620.35	29.02	12.52
A-5-79	83.28	0.04	1.05	1.42	0.26	444.41	19.45	10.31

# YET MORE NUMERICAL RESULTS

	DET	TR	PWL3	PWL3+TR	PWQ3	H50	TR-H7	DET+TR-H7
A-5-39	0.42	0.05	0.02	0.04	0.01	0.00	0.06	0.13
A-5-12	0.49	1.75	0.40	1.74	*	0.02	0.04	0.00
A-5-48	0.49	0.13	0.03	0.03	0.03	0.01	0.09	0.14
A-5-46	0.69	0.58	0.06	0.43	0.03	0.02	0.17	0.16
A-5-28	0.52	0.10	0.03	0.10	0.01	0.14	0.05	0.15
A-5-79	0.48	0.06	0.02	0.04	0.01	0.00	0.08	0.12