A Stackelberg Game Theoretic Approach for Urban Freight Transportation

Tao Yao

The Pennsylvania State University

Joint work with Terry L. Friesz, Bo Zhang

INFORMS TSL Workshop, June 26-29, 2011

Outline

- Introduction
- Literature Review
- Modeling Framework
- Dynamic User Equilibrium (DUE)
- Model Formulation
- Solution Methodology
- Conclusion and Future Study

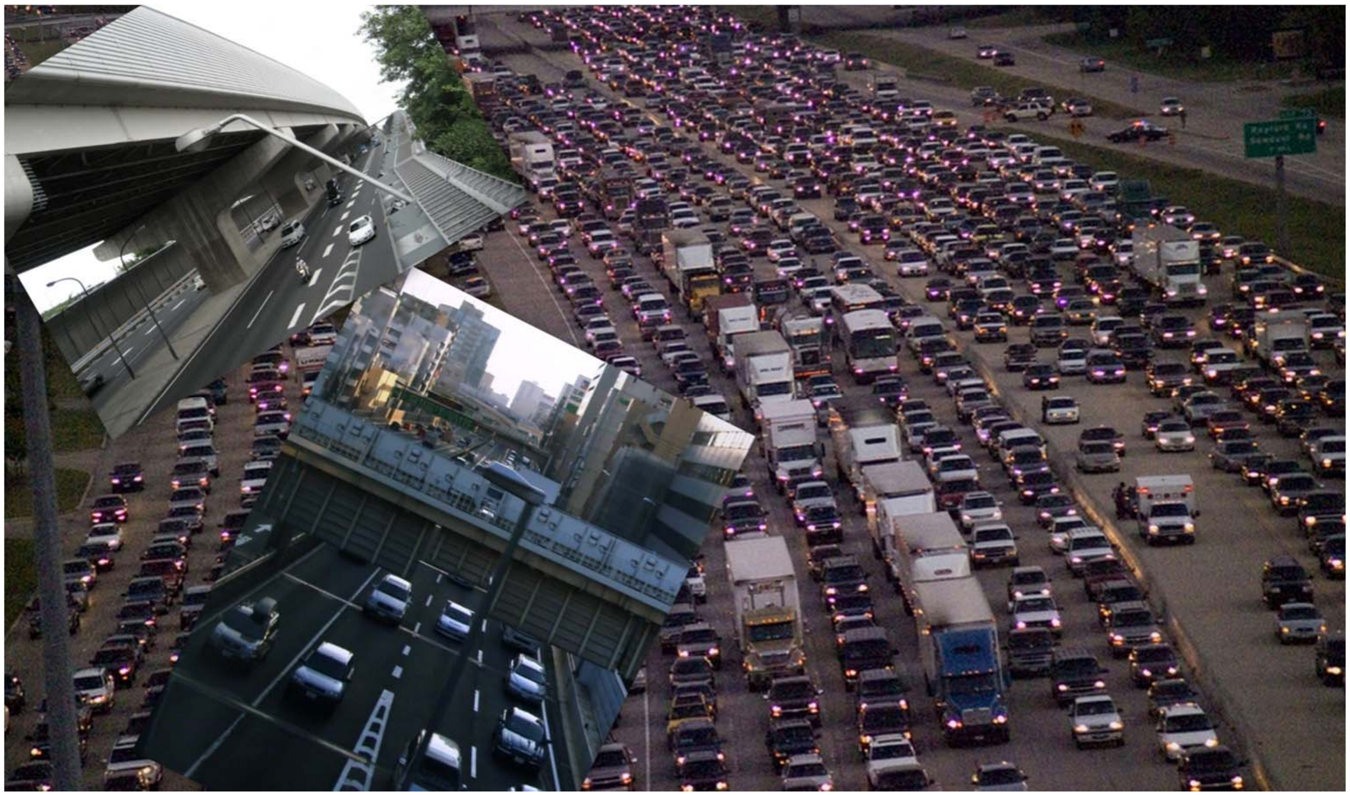
Introduction

- Urban freight transportation aims to reduce the negative externalities, such as emission, noise and congestion, associated to the freight activity while supporting their economic and social development

- Challenges due to economic and environmental factors:
  - Growing number of private vehicles
  - Growing demand of urban freight transportation service
  - Recognition of the need for a paradigm shift toward environmentally sustainable logistics and freight technologies that create a good-neighbor image

Introduction

- Congestion



http://www.whatmattersweblog.com/2009/03/08/the-fundamental-transition-of-the-auto-industry/

Introduction

- Motivation
  - Previous studies focus on modeling the behaviors of shippers and carriers of the urban freight. However little has considered the behaviors of private vehicles transporting people while modeling the urban freight transportation problem (Ambrosini and Routhier 2004)

- Objective
  - Investigate the interaction between urban freight transportation and the behaviors of private vehicles transporting people

Introduction

- This talk is primarily about the formulation of a differential Stackelberg game describing an urban freight carrier's selection of
  - departure time, and
  - route
- The setting is an urban metropolis whose road network experiences congestion.
- Algorithmic/computational research is just now beginning and not reported here.

Literature Review

- City Logistics and Vehicle Routing
  - Taniguchi and Thompson (2002) Modeling city logistics
  - Crainic et al. (2009) Models for evaluating and planning city logistics systems
  - Kulkarni and Bhave (1985) Integer programming formulations of vehicle routing problems
  - Laporte (1992) The vehicle routing problem: an overview of exact and approximate algorithms
  - Bell (2004) Games, heuristics, and risk averseness in vehicle routing problems
  - Kallehauge et al. (2005) Vehicle routing problem with time windows
  - Qureshi et al. (2009) An exact solution approach for vehicle routing and scheduling problems with soft time windows

Literature Review

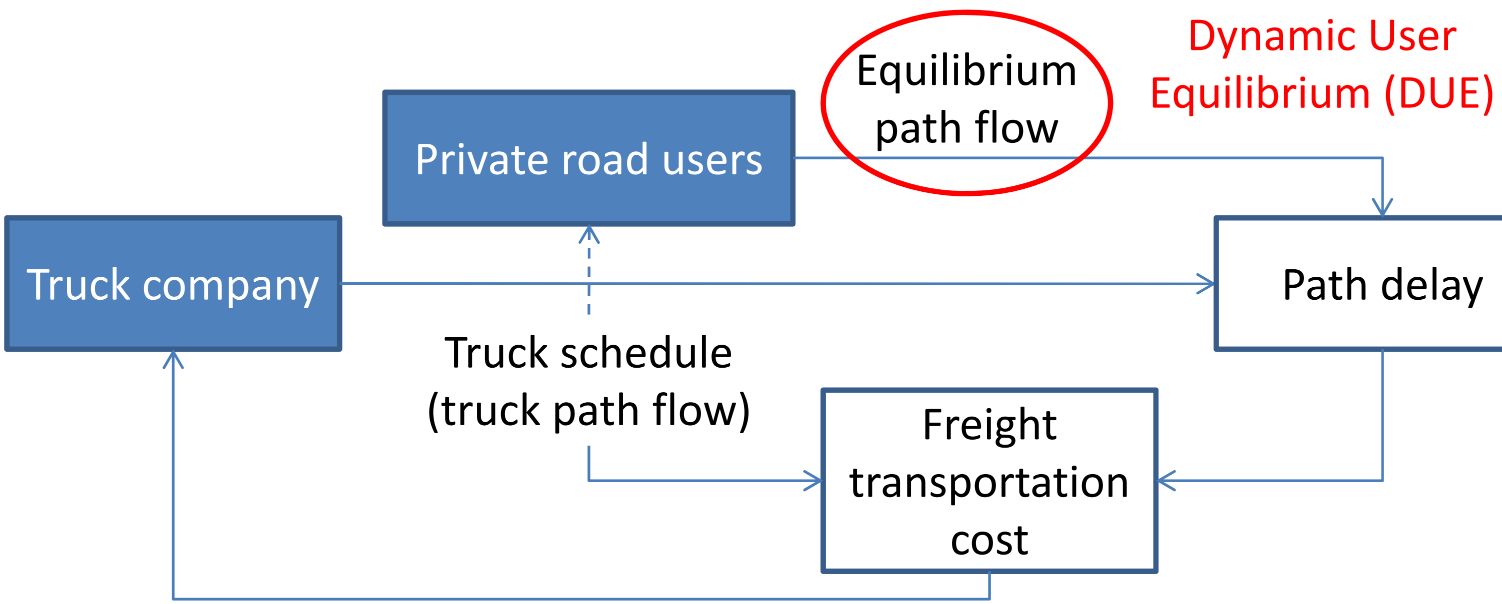
- Freight Network Flow Models
  - Friesz et al. (1983) Predictive intercity freight network models: the state of the art
  - Friesz et al. (1985) Economic and computational aspects of freight network equilibrium models: a synthesis
  - Friesz et al. (1986) A Sequential shipper-carrier network model for predicting freight flows
  - Harker and Friesz (1986) Prediction of intercity freight flows
  - Agrawal and Ziliaskopoulos (2006) Shipper-carrier dynamic freight assignment model using a variational inequality approach

Literature Review

- Dynamic Network Traffic Equilibrium
  - Friesz et al. (1993) A variational inequality formulation of the dynamic network traffic equilibrium problem
  - Friesz et al. (2001) Dynamic network user equilibrium with state-dependent time lags
  - Friesz and Mookherjee (2006) Solving the dynamic network user equilibrium problem with state-dependent time shifts

Modeling Framework

- A Stackelberg Game
  - A strategic game in economics in which the leader firm moves first and then the follower firms move sequentially
  - Leader: a truck company
  - Follower: all private road users
  - Mathematical Programming with Equilibrium Constraints (MPEC)



Dynamic User Equilibrium (DUE)

Dynamic User Equilibrium (DUE) is one type of Dynamic Traffic Assignment (DTA) wherein the effective unit travel delay, including early and late arrival penalties, of travel for the same purpose is identical for all utilized path and departure time pairs

- Traffic Dynamic Models. Merchant and Nemhauser (1978a,b), Ran et al. (1993) and Ran and Boyce (1996)
- Friesz et. al (1993) proposed a variational inequality (VI) formulation that is a dynamic generalization of the static Wardropian user equilibrium
- Friesz et. al (2001) formulated DUE problem as differential variational inequality (DVI) with state-dependent time shift.
- Friesz et. al (2010) approximated dynamic network loading by using the second order Taylor expansion for computational efficiency.

Dynamic User Equilibrium (DUE)

- Model Components (Peeta and Ziliaskopoulos (2001))
  - Model of path delay
  - Arc dynamics
  - Flow propagation constraints
  - Flow conservation and nonnegativity
  - Route and departure time choice model

Dynamic User Equilibrium (DUE)

- Basic Notation
  - The interval of continuous time:

$$\left[t_0, t_f\right] \subset \Re_+^1$$

  - The path unit delay operator for path p:

$$D_p\left(t, h\right) \quad \forall p \in \mathrm{P}$$

where

P is the set of all paths employed by travelers

t denotes departure time

h is a vector of departure rates (“path flows”)

Dynamic User Equilibrium (DUE)

- Effective Delay Operators
  - The effective unit path delay operators $\Psi_p(t,h)$ are formed by adding schedule delay $F(\cdot)$ to path delay $D_p(t,h)$:

$$\Psi_p(t,h) = D_p(t,h) + F\left[t + D_p(t,h) - T_A\right] \quad \forall p \in P$$

where

$T_A =$ is the desired arrival time

$T_A < t_f$

The function $F(\cdot)$ assesses a penalty whenever

$$t + D_p(t,h) \neq T_A$$

Dynamic User Equilibrium (DUE)

- Flow Conservation
  - Fixed trip matrix

$$Q = \left(Q_{ij} : (i, j) \in W \right)$$

where

$Q_{ij} \in \Re^{1}_{++}$ is the fixed travel demand, origin-destination pair $(i, j) \in W$

$W =$ the set of all origin-destination pairs

  - We stipulate that

$$h \in \left(L^{2}_{+} \left[t_0, t_f \right] \right)^{|P|}$$

we write the flow conservation constraint as

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} h_p(t)\, dt = Q_{ij} \quad \forall (i, j) \in W$$

where

$P_{ij} =$ subset of paths that connect origin-destination pair $(i, j) \in W$

Dynamic User Equilibrium (DUE)

- DUE Defined
  - We define the set of feasible flows by

$$\Lambda_0 = \left\{ h \geq 0 : \sum_{p \in P_{ij}} \int_{t_0}^{t_f} h_p(t)\,dt = Q_{ij} \quad \forall (i,j) \in W \right\} \subset \left(L_+^2 \left[t_0, t_f \right] \right)^{|P|}$$

  - Per Friesz et al. (1993), we define equilibrium as follows:

A vector of departure rates (path flows) $h^* \in \Lambda_0$ is a dynamic user equilibrium if

$$h_p^*(t) > 0,\ p \in P_{ij} \Rightarrow \Psi_p\left[t, h^*(t)\right] = v_{ij}$$

We denote this equilibrium by $DUE\left(\Psi, \Lambda_0, t_0, t_f\right)$

Dynamic User Equilibrium (DUE)

- DVI Formulation
  - The DUE problem can be expressed (Friesz et al (2010)) as the following DVI:

$$\left.\begin{array}{l} \text{find } h^* \in \Lambda \text{ such that} \\ \sum_{p\in P} \int_{t_0}^{t_f} \Psi_p\left(t,h^*\right)\left(h_p - h_p^*\right)dt \geq 0 \\ \forall h \in \Lambda \end{array}\right\} DVI\left(\Psi,\Lambda,t_0,t_f\right)$$

where

$$\Psi = \text{the effective delay operator}$$

$$\Lambda = \left\{\begin{array}{l} h \geq 0 : \dfrac{dy_{ij}}{dt} = \sum_{p\in P_{ij}} h_p\left(t\right),\ y_{ij}\left(t_0\right) = 0,\ y_{ij}\left(t_f\right) = Q_{ij} \\ \forall \left(i,j\right) \in W \end{array}\right\}$$

$$Q = \left(Q_{ij}\right) = \text{the trip matrix}, \quad t_0 = \text{start time}, \quad t_f = \text{end time}$$

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: More Notation
  - Path:

    $$p = \{a_1, a_2, \ldots, a_i, \ldots, a_{m(p)}\}$$

  - Path flows:

    $$h_p$$

  - Arc exit flows:

    $$g_{a_i}^p$$

  - Traffic volume on an arc for a path:

    $$x_{a_i}^p$$

  - Definition:

    Find arc exit flows (g) and arc volumes $(x) =$ network loading

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: Defining Path Delay for the Point Queue Model (PQM)
  - Assume an arc delay function:

$$D_{a_i}\left(x_{a_i}(t)\right)$$

  - Note:

$$\pi_{a_i}^{p}(t)=\pi_{a_{i-1}}^{p}(t)+D_{a_i}\left[x_{a_i}\left(\pi_{a_{i-1}}^{p}(t)\right)\right]$$

  - Total path delay:

$$D_p(t)=\sum_{i=1}^{m(p)}\left[\pi_{a_i}^{p}(t)-\pi_{a_{i-1}}^{p}(t)\right]=\pi_{a_{m(p)}}^{p}(t)-t$$

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: Nested Path Delay Operators
  - Nested path delay operators:

$$D_p(t,x) = \sum_{i=1}^{m(p)} \delta_{a_i p} \Phi_{a_i}(t,x)$$

where

$$\delta_{ap} = \begin{cases} 1 & \text{if arc a belongs to path p} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\Phi_{a_1}(t,x) = D_{a_1}\left(x_{a_1}(t)\right)$$

$$\Phi_{a_2}(t,x) = D_{a_2}\left(x_{a_2}\left(t + \Phi_{a_1}(t,x)\right)\right)$$

$$\vdots$$

$$\Phi_{a_i}(t,x) = D_{a_i}\left(x_{a_i}\left(t + \sum_{j=1}^{i-1} \Phi_{a_j}(t,x)\right)\right)$$

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: Arc Dynamics
  - State dynamics:

$$\frac{dx_{a_1}(t)}{dt} = h_p(t) - g^p_{a_1}(t) \quad \forall p \in \mathrm{P}$$

$$\frac{dx_{a_i}(t)}{dt} = g^p_{a_{i-1}}(t) - g^p_{a_i}(t) \quad \forall p \in \mathrm{P}, i \in [2, m(p)]$$

$$x^p_{a_i}(t_0) = x^p_{a_i,0} \quad \forall p \in \mathrm{P}, i \in [1, m(p)]$$

  - Approximation , precise only in the limit of infinitesimal arc length.
  - Total arc volume:

$$x_0(t) = \sum_{p \in P} \delta_{ap} x^p_a(t) \quad \forall a \in A$$

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: Flow Propagation Constraints

  – Flow propagation constraints:

$$g_{a_i}^{p}\left(t+D_{a_i}\left[x_{a_i}(t)\right]\right)\left(1+\dot{D}_{a_i}\left[x_{a_i}(t)\right]\cdot\dot{x}_{a_i}(t)\right)=g_{a_{i-1}}^{p}(t)$$

  – FIFO requires

$$1+\dot{D}_{a_i}\left[x_{a_i}(t)\right]\cdot\dot{x}_{a_i}(t)>0$$

Dynamic User Equilibrium (DUE)

- The Network Loading Phase: Differential Algebraic Equation (DAE) System for the PQM

$\forall p \in \mathrm{P}, i \in [1, m(p)]$

$$\frac{dx_{a_1}(t)}{dt} = h_p(t) - g_{a_1}^p(t)$$

$$\frac{dx_{a_i}(t)}{dt} = g_{a_{i-1}}^p(t) - g_{a_i}^p(t)$$

$$x_{a_i}^p(t_0) = x_{a_i,0}^p$$

$$g_{a_{i-1}}^p(t) = g_{a_i}^p\left(t + D_{a_i}\left[x_{a_i}(t)\right]\right)\left(1 + \dot{D}_{a_i}\left[x_{a_i}(t)\right] \cdot \dot{x}_{a_i}(t)\right)$$

$$h_p(t) = g_{a_1}^p\left(t + D_{a_1}\left[x_{a_1}(t)\right]\right)\left(1 + \dot{D}_{a_1}\left[x_{a_1}(t)\right] \cdot \dot{x}_{a_1}(t)\right)$$

Model Formulation

- Notation
  - Consistent with those in DUE
  - New terms:

$y = [y_{a_i}]$: the vector of truck volumes on all arcs

$\tau = [\tau_p]$: the vector of truck flows on all paths

$g = [g_{a_i}^{p,truck}]$: the vector of link exit (entrance) truck flows

$g = [g_{a_i}^{p,auto}]$: the vector of link exit (entrance) other automobile flows (in contrast to truck flows)

$Q = [Q_{ij}^{truck}]$: the vector of truck demand

$Q = [Q_{ij}^{auto}]$: the vector of other automobile demand (in contrast to truck demand)

Model Formulation

- Objective function: minimizing the total truck travel cost

$$\min \sum_{p \in P} \int_{t_0}^{t_f} \Psi_p\left(t, h^*(t), \tau(t)\right) \tau_p(t) dt \qquad (8)$$

- Demand and nonnegativity constraints

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} h_p(t) dt = Q_{ij}^{auto}, \quad h_p(t) \geq 0 \quad \forall (i,j) \in W \qquad (9)$$

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} \tau_p(t) dt = Q_{ij}^{truck}, \quad \tau_p(t) \geq 0 \quad \forall (i,j) \in W \qquad (10)$$

- DVI for private vehicle flows

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t, h^*(t), \tau(t)\right) \left[h_p(t) - h_p^*(t) \right] dt \geq 0 \qquad (11)$$

for all *h* that satisfy the Differential Algebraic Equation (DAE) system

Model Formulation

- DAE system

$$\frac{dx_{a_1}^p(t)}{dt} = h_p(t) - g_{a_1}^{p,auto}(t) \quad \forall p \in \mathrm{P} \tag{12}$$

$$\frac{dy_{a_1}^p(t)}{dt} = \tau_p(t) - g_{a_1}^{p,truck}(t) \quad \forall p \in \mathrm{P} \tag{13}$$

$$\frac{dx_{a_i}^p(t)}{dt} = g_{a_{i-1}}^{p,auto}(t) - g_{a_i}^{p,auto}(t) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \tag{14}$$

$$\frac{dy_{a_i}^p(t)}{dt} = g_{a_{i-1}}^{p,truck}(t) - g_{a_i}^{p,truck}(t) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \tag{15}$$

$$h_p(t) = g_{a_1}^{p,auto}\left(t + D_{a_1}\left[x_{a_1}(t) + y_{a_1}(t)\right]\right)\left(1 + D'_{a_1}\left[x_{a_1}(t) + y_{a_1}(t)\right]\left(\dot{x}_{a_1}(t) + \dot{y}_{a_1}(t)\right)\right) \tag{16}$$

$$g_{a_{i-1}}^{p,auto}(t) = g_{a_i}^{p,auto}\left(t + D_{a_i}\left[x_{a_i}(t) + y_{a_i}(t)\right]\right)\left(1 + D'_{a_i}\left[x_{a_i}(t) + y_{a_i}(t)\right]\left(\dot{x}_{a_1}(t) + \dot{y}_{a_1}(t)\right)\right) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \tag{17}$$

$$\tau_p(t) = g_{a_1}^{p,truck}\left(t + D_{a_1}\left[x_{a_1}(t) + y_{a_1}(t)\right]\right)\left(1 + D'_{a_1}\left[x_{a_1}(t) + y_{a_1}(t)\right]\left(\dot{x}_{a_1}(t) + \dot{y}_{a_1}(t)\right)\right) \tag{18}$$

$$g_{a_{i-1}}^{p,truck}(t) = g_{a_i}^{p,truck}\left(t + D_{a_i}\left[x_{a_i}(t) + y_{a_i}(t)\right]\right)\left(1 + D'_{a_i}\left[x_{a_i}(t) + y_{a_i}(t)\right]\left(\dot{x}_{a_1}(t) + \dot{y}_{a_1}(t)\right)\right) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \tag{19}$$

$$x(t_0) = x_0 \tag{20}$$

$$y(t_0) = y_0 \tag{21}$$

Solution Methodology

- For the lower-level DUE, reformulation is needed
  - Reformulation proposed by Tan et al. (1979)
  - Nonlinear complementary problem (NCP) reformulation
  - Fixed point reformulation
- Algorithm can be used to solve the whole MPEC
  - Reformulate as single-level NLP, and apply various approaches for NLP.
  - Heuristic Method (Simulated Annealing (SA), Particle Swarm Optimization (PSO))

Solution Methodology

- NCP Reformulation of DUE

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} F_p\left(t, h^*(t), \tau(t)\right)\left[h_p(t) - h_p^*(t)\right] dt \geq 0 \qquad (11)$$

↓

$$F_p\left(t, h^*(t), \tau(t)\right) + \lambda_{ij}(t) - \rho_p(t) = 0 \quad \forall p \in P_{ij} \qquad (22)$$

$$\rho_p(t) h_p(t) = 0 \quad \forall p \in P_{ij} \qquad (23)$$

$$\rho_p(t) \geq 0 \quad \forall p \in P_{ij} \qquad (24)$$

Solution Methodology

- An Iterative Algorithm
  - Step 0. Initialization.
    Identify an initial feasible solution h^0, τ^0 and set $k = 0$
  - Step 1. Network Loading
    Calculate the solution of the discrete version of the DAE system for
    $D_{a_i}\left(x_{a_i}(t)\right)$
  - Step 2. Compute dual variable: $\lambda_{ij}(t)$

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} \left[h_p^k(t) + \lambda_{ij}^{k+1}(t) - \eta \Psi_p\left(t, h^k(t), \tau^k(t)\right) \right]_+ dt = Q_{ij} \quad \forall (i,j) \in W$$

Solution Methodology

- An Iterative Algorithm (cont'd)
  - Step 3: Solve the discrete version of the following MPCP problem using $\lambda_{ij}(t)$ and $D_{a_i}\left(x_{a_i}(t)\right)$ as inputs and call the solution h^{k+1}, τ^{k+1}

$$\min \sum_{p \in \mathrm{P}} \int_{t_0}^{t_f} \Psi_p\left(t, h^*(t), \tau(t)\right) \tau_p(t) dt$$

$$\Psi_p\left(t, h^*(t), \tau(t)\right) + \lambda_{ij}(t) - \rho_p(t) = 0 \quad \forall p \in P_{ij}$$

$$\rho_p(t) h_p(t) = 0 \quad \forall p \in P_{ij}$$

$$\rho_p(t) \geq 0 \quad \forall p \in P_{ij}$$

$$\sum_{p \in \mathrm{P}_{ij}} \int_{t_0}^{t_f} h_p(t) dt = Q_{ij}^{auto}, \quad h_p(t) \geq 0 \quad \forall (i,j) \in W$$

$$\sum_{p \in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \tau_p(t) dt = Q_{ij}^{truck}, \quad \tau_p(t) \geq 0 \quad \forall (i,j) \in W$$

  - Step 4: Stop if $| h^{k+1} - h^k | \leq \varepsilon$

Conclusion and Future Study

- Conclusion
  - We have built a Dynamic MPEC to study the interaction between urban freight transportation and private vehicle flow

- Future Study
  - Conduct numerical experiments and interpret the result
  - Provide guidelines for truck company to assist truck scheduling
  - Formulate the upper level of the MPEC as a vehicle routing problem (VRP) and develop methods to solve the new problem

Thank you

- Questions?