Multicriteria Air Traffic Flow Management

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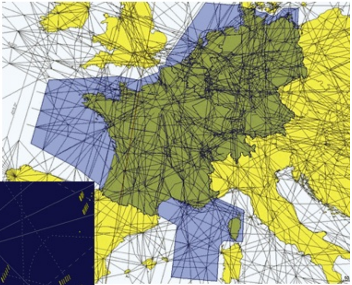
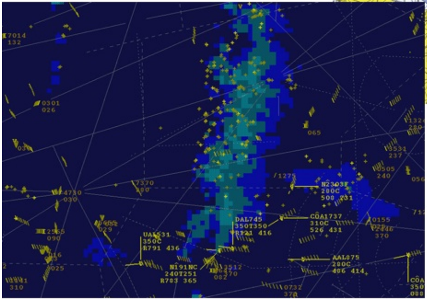
Outline

Airport arrival rate

Airspace capacity





ATFM formulation

$$x_{f,k,t} = \begin{cases} 1 & \text{if flight } f \text{ has} \\ & \text{departed from / flown through / arrived at} \\ & \text{airport / airspace sector } k \\ & \text{before time } t \\ 0 & \text{otherwise} \end{cases}$$

ATFM formulation

$$\min \sum_{f \in F} \Big[(-c_f^a) \sum_{t \in T_{f,P(f,N_f)}} x_{f,P(f,N_f),t} + (c_f^a - c_f^g) \sum_{t \in T_{f,P(f,1)}} x_{f,P(f,1),t}\Big]$$

$$\sum_{f:P(f,1)=k} (x_{f,k,t} - x_{f,k,t-1}) \leq D_{k,t} \qquad \forall k \in \mathcal{A}, t \in \mathcal{T}$$

$$\sum_{f:P(f,N_f)=k} (x_{f,k,t} - x_{f,k,t-1}) \leq A_{k,t} \qquad \forall k \in \mathcal{A}, t \in \mathcal{T}$$

$$\sum_{f:P(f,i)=k, i<N_f} (x_{f,k,t} - x_{f,P(f,i+1),t}) \leq S_{k,t} \qquad \forall k \in \mathcal{S}, t \in \mathcal{T}$$

$$x_{f,k,t-1} - x_{f,k,t} \leq 0 \qquad \forall f \in F, k \in \rho_f, t \in T_{f,k}$$

$$x_{f,P(f,i),t} - x_{f,P(f,i-1),t-\beta_{f,P(f,i-1)}} \leq 0 \qquad \forall f \in F, 2 \leq i \leq N_f, t \in T_{f,P(f,i)}$$

$$x_{f_2,P(f_2,1),t} - x_{f_1,P(f_1,N_{f_1}),t-\chi_{f_2}} \leq 0 \qquad \forall (f_1,f_2) \in \mathcal{C}, t \in T_{f_2,P(f_2,1)}$$

Efficiency and equity

“a primary objective of the [Federal Aviation Administration’s Air Traffic Management] functions is to provide fair and equitable access” (Vossen et al., 2010)

an air traffic flow management problem “formulation should essentially consider a bi-criterion approach, enabling the efficient study of the trade-off curve between” equity and efficiency (Fearing et al., 2009)

Environmental impacts

Increasingly, we are concerned about environmental impacts

- Noise
- Fuel use
- NO_x
- SO_x
- CO
- HC

Air traffic flow management decisions have large environmental impacts (Carlier et al., 2007)

A multicriteria approach

The goal of this research is to show the trade-offs between

- Efficiency
- Equity
- Environmental impacts

We want to get a feel for the Pareto frontier.

Do we need to find each and every point on the frontier?

Outline

The weighted sum method

$$\min \quad (\lambda)z_1 + (1-\lambda)z_2$$

$$\text{s.t.} \quad ....$$

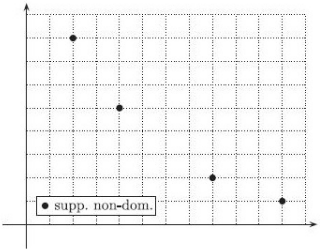
Uses a composite objective function which is a linear combination of distinct objectives

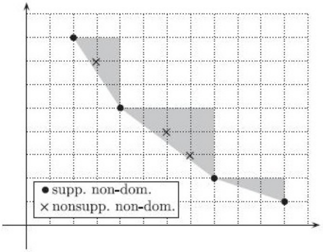
(penalize inequity, price emissions)

Solutions to multicriteria problem

Supported non-dominated solutions
Points on boundary of convex hull of feasible region (in objective space)

Nonsupported non-dominated solutions
All other Pareto-optimal policies





The weighted sum method

Correct: Returns non-dominated solutions
Computable: Does not break problem structure
but...
Not Complete: Finds only supported non-dominated solutions
May not show trade-off between objective functions

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also...
You must select and parameterize a model combining objective functions.
Arbitrarily small changes in parameter values can lead to arbitrarily large changes in policy.

Two-phase methods

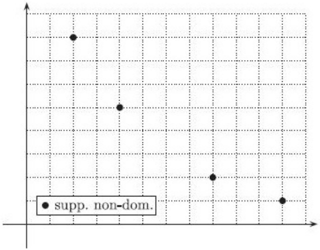
Phase 1: Find all supported efficient solutions

Phase 2: Find unsupported efficient solutions

Phase 1

The Dichotomic Approach

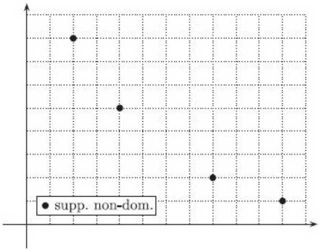
- Apply lexicographic optimization to get 2 solutions
- If two solutions are distinct, there is a gap
- If there is a gap, use the weighted sum method
- Repeat

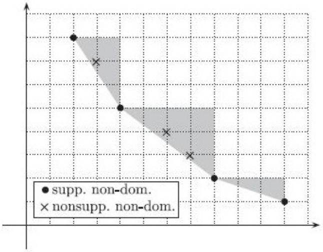


Phase 2

We can now search for solutions in gaps left by Phase 1

Use elastic-constraint or other method to fill in gaps



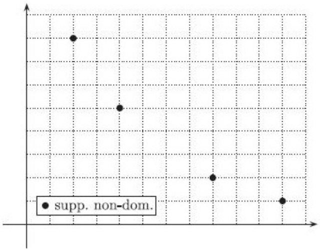


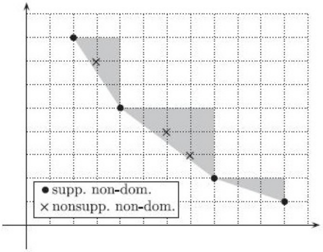
The ϵ-constraint method

Start by optimizing with respect to objective 1

Constrain value of objective 2 to slightly less than current value

Resolve, if no solution exists STOP, otherwise Return to step 2





The elastic constraint method

$$\begin{aligned} \min \quad & z_1 + (\lambda)s \\ \text{s.t.} \quad & z_2 - s \leq \epsilon \\ & s \geq 0 \\ & \ldots. \end{aligned}$$

The ϵ-cover method

Time could still be an issue.

We can stop the algorithm after a set amount of time.

Or we could use a relatively quick ϵ-cover approach.
    Find a set of solutions that “cover” the Pareto frontier.

My conclusion: two-phase methods are ideal for ATFM

We find the delay cost minimizing solution in the same amount of time as single-objective formulations.

We find supported efficient solutions in the same amount of time as the weighted sum method.

We find other Pareto-optimal solutions as time allows.

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same amount of time as single-objective formulations.
We find supported efficient solutions in the same amount
of time as the weighted sum method.
We find other Pareto-optimal solutions as time allows.

We show trade-offs / many Pareto-optimal policies.
(Weighted sum methods may not.)

We don't have to come up with perfect models for the
price of inequity or emissions.

Outline

Reversals

Ration by Schedule (RBS) - flights are granted access to capacity constrained airports in a way that preserves the order of initial schedules.

A “reversal” occurs when the order with which a pair of flights is initially scheduled to use a capacity constrained airport is reversed. (Gupta and Bertsimas, 2010)

Minimizing delay costs and reversals

$$\min \sum_{f \in F} \left[(-c_f^a) \sum_{t \in T_{f,P(f,N_f)}} x_{f,P(f,N_f),t} + (c_f^a - c_f^g) \sum_{t \in T_{f,P(f,1)}} x_{f,P(f,1),t} \right]$$

$$\min \sum_{(f_1,f_2) \in \mathcal{R}} r_{f_1,f_2}$$

$$\sum_{f:P(f,1)=k} (x_{f,k,t} - x_{f,k,t-1}) \leq D_{k,t} \qquad \forall k \in \mathcal{A}, t \in \mathcal{T}$$

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$$x_{f_2,P(f_2,1),t} - x_{f_1,P(f_1,N_{f_1}),t-\chi_{f_2}} \leq 0 \qquad \forall (f_1,f_2) \in \mathcal{C}, t \in T_{f_2,P(f_2,1)}$$

$$x_{f_2,P(f_2,N_{f_2}),t} - x_{f_1,P(f_1,N_{f_1}),t} - r_{f_1,f_2} \leq 0 \qquad \forall (f_1,f_2) \in \mathcal{R}, t \in T_{f_2,P(f_2,N_{f_2})}$$

Minimizing delay costs and environmental impacts

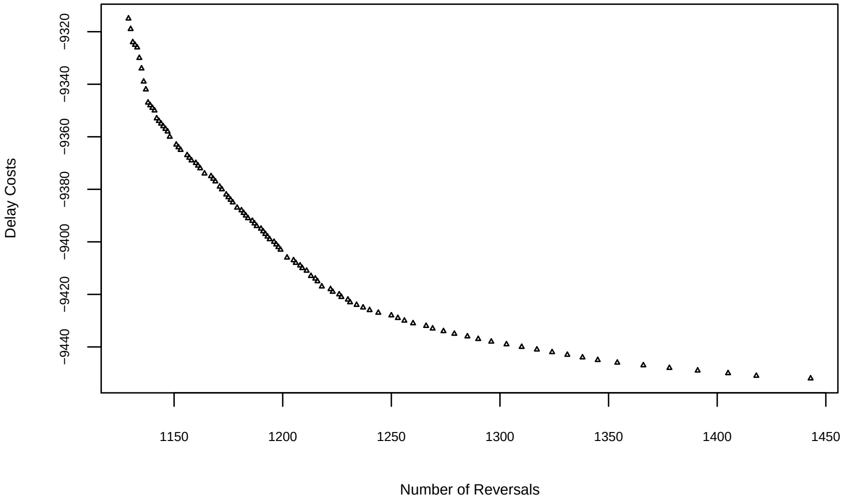
$$\min \sum_{f\in F}\Big[(-c^a_f)\sum_{t\in T_{f,P(f,N_f)}} x_{f,P(f,N_f),t} + (c^a_f - c^g_f)\sum_{t\in T_{f,P(f,1)}} x_{f,P(f,1),t}\Big]$$

$$\min \sum_{f\in F}(e^a_f)\Big[\sum_{t\in T_{f,P(f,N_f)}} t(x_{f,P(f,N_f),t} - x_{f,P(f,N_f),t-1}) - \sum_{t\in T_{f,P(f,1)}} t(x_{f,P(f,1),t} - x_{f,P(f,1),t-1})\Big] +$$

$$(e^g_f)\Big[\sum_{t\in T_{f,P(f,1)}} t(x_{f,P(f,1),t} - x_{f,P(f,1),t-1}) - E_{f,1}\Big]$$

[... as before ...]

Sample trade-off curve



Conclusion

Found evidence of competing objectives for ATFM

Suggested formal multicriteria optimization is appropriate
- A two-phase approach appears best

Further research needed to improve formulations
- Realism
- Computational concerns