Trial-and-Error Implementation of Tradable Credits Scheme with Unknown Demand Function

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Outline

- Introduction to tradable credit schemes
- Trial-and-error implementation of congestion pricing/tradable credit schemes with unknown demand function
    - Non-convergence and modification of bisection methods for a single road
    - Extension to a general network
- Conclusion and extension

Pricing and Quantity Instruments for Network Mobility Management

- Pricing Instrument
  - Efficient utilization of limited network resources
  - Inequality problem etc. → public opposition
- Quantity Instrument
  - Road space rationing (Beijing)
  - Tradable driving permit/credit scheme

Introduction to Tradable Travel Credit Schemes

- Each participating agent receives a proportion of credits (on a periodic basis such a month or a quarter);
  - Equity
- Initial distribution for free;
  - Revenue neutral incentives for mobility and environmental quality
- Credit charging scheme:
  - Link-specific or cordon-based; distance or time-based; time-invariant or time-varying

Introduction to Tradable Travel Credit Schemes

- A policy target is defined in terms of fix-quantity driving credits;
  - ✓ Easily achieve the policy target

    Example: Distance-based credit charge for achieving control of total veh-km traveled on the network

- The equilibrium price of credits is determined by the market through free trading.
  - ✓ Market driven
  - ✓ Enhance income distribution or financial transfer confined only to within the predefined group of travelers (credit flow from the higher income groups to the lower; the flow of money from the wealthy to the less)

Recent Study:

Yang, H. and Wang, X.L. (2011) Managing network mobility with tradable credits. Transportation Research **45B (3) 580-594.**

- Proposed and formulated such a tradable credit scheme in general networks with either fixed or elastic demand.
- For a given credit scheme, a unique equilibrium flow pattern exists; the equilibrium credit price is unique subject only to very mild assumptions.
- A properly designed tradable credit scheme (total amount of credits issued and individual link credit charges) can emulate a congestion pricing system and support various desirable traffic flow optima:
  - ➢ Social optimum
  - ➢ Pareto-improving and revenue-neutral
  - ➢ Capacity constrained traffic flow pattern

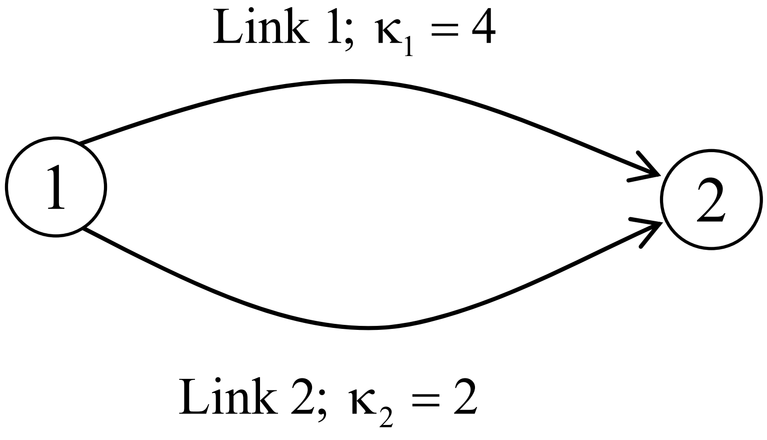
Traffic Equilibrium and Market Equilibrium with Tradable Credits: An Example

Link travel time: $t_1(v_1) = 8 + 2v_1$, $t_2(v_2) = 16 + v_2$

O-D demand: $d_{1\to 2} = 10$,

Initial credit allocation: $K = 30$, $k_{1\to 2} = 3$

Link credit charge: $\kappa_1 = 4$, $\kappa_2 = 2$.



User equilibrium and credit market equilibrium conditions:

$$8 + 2v_1 + 4p = 16 + v_2 + 2p$$

$$v_1 + v_2 = 10$$

$$4v_1 + 2v_2 = 30$$

A unique equilibrium solution:

Link flow: $v_1^* = v_2^* = 5$

Link travel time: $t_1^* = 18$, $t_2^* = 21$

Unit credit price; $p^* = 1.5$

Implementation Issue under Limited Information

(for both pricing and tradable credit schemes)

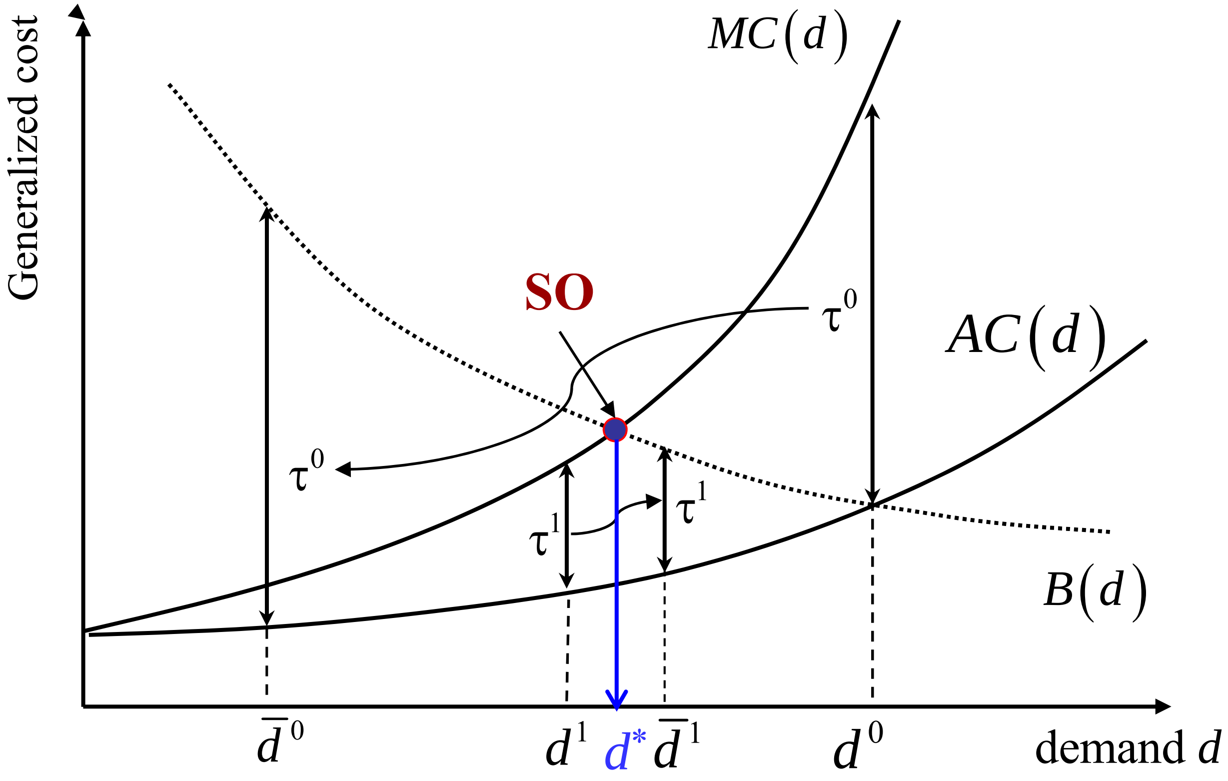
- Analytical demand functions tailored for traffic demand control are difficult to establish in practice even with advanced transport modeling techniques.
  - *Individual users' valuation of trip-making (hidden information)*
  - *Anonymous individual users' behavior (hidden action)*
- A fundamental question: *how to design the optimal credit scheme (credit allocation and charge) in a simple yet practical manner ?*

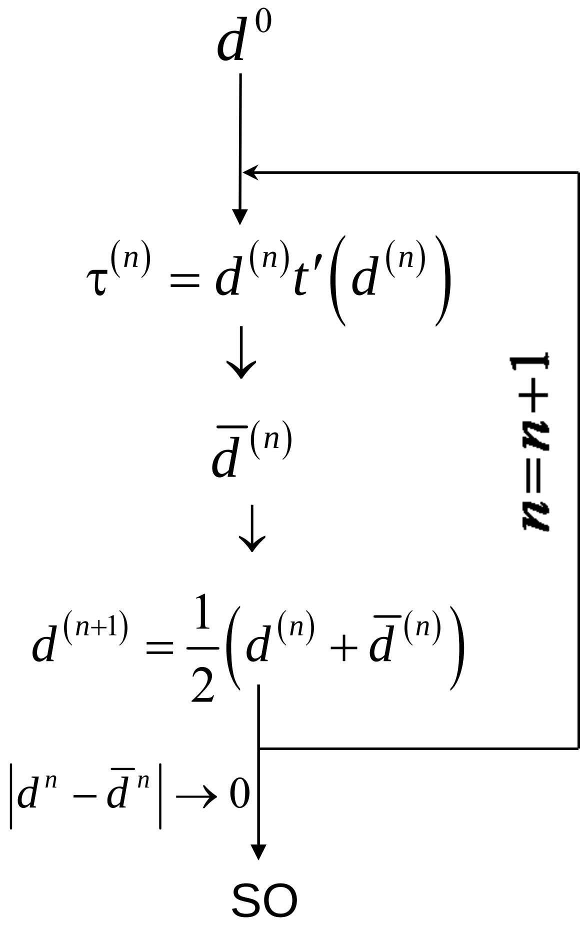
Bisection Method for Trial-and-Error Implementation of Marginal Cost Pricing on a Single Road

Li, M. Z. F., 2002. The role of speed-flow relationship in congestion pricing implementation with an application to Singapore. Transportation Research 36B (8), 731-754.

Trial d^n and observed $\bar{d}^n$ is always on different sides of the unknown optimal d^*

$$\left|d^{(n+1)} - \bar{d}^{(n+1)}\right| < \frac{\left|d^{(n)} - \bar{d}^{(n)}\right|}{2}$$





$$d^0$$

$$\tau^{(n)} = d^{(n)} t'\left(d^{(n)}\right)$$

$$\downarrow$$

$$\bar{d}^{(n)}$$

$$\downarrow$$

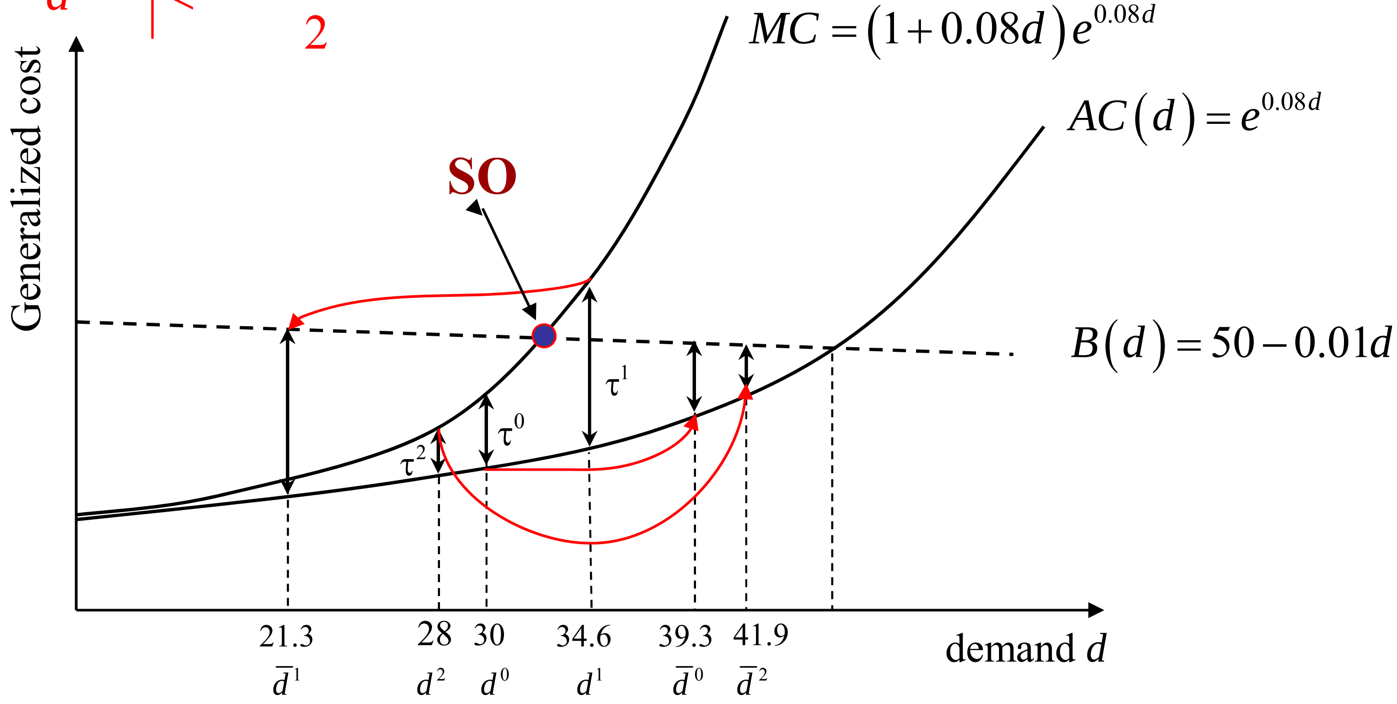
$$d^{(n+1)} = \frac{1}{2}\left(d^{(n)} + \bar{d}^{(n)}\right)$$

$$\left|d^n - \bar{d}^n\right| \to 0$$

SO

Non-convergence of the Bisection Method

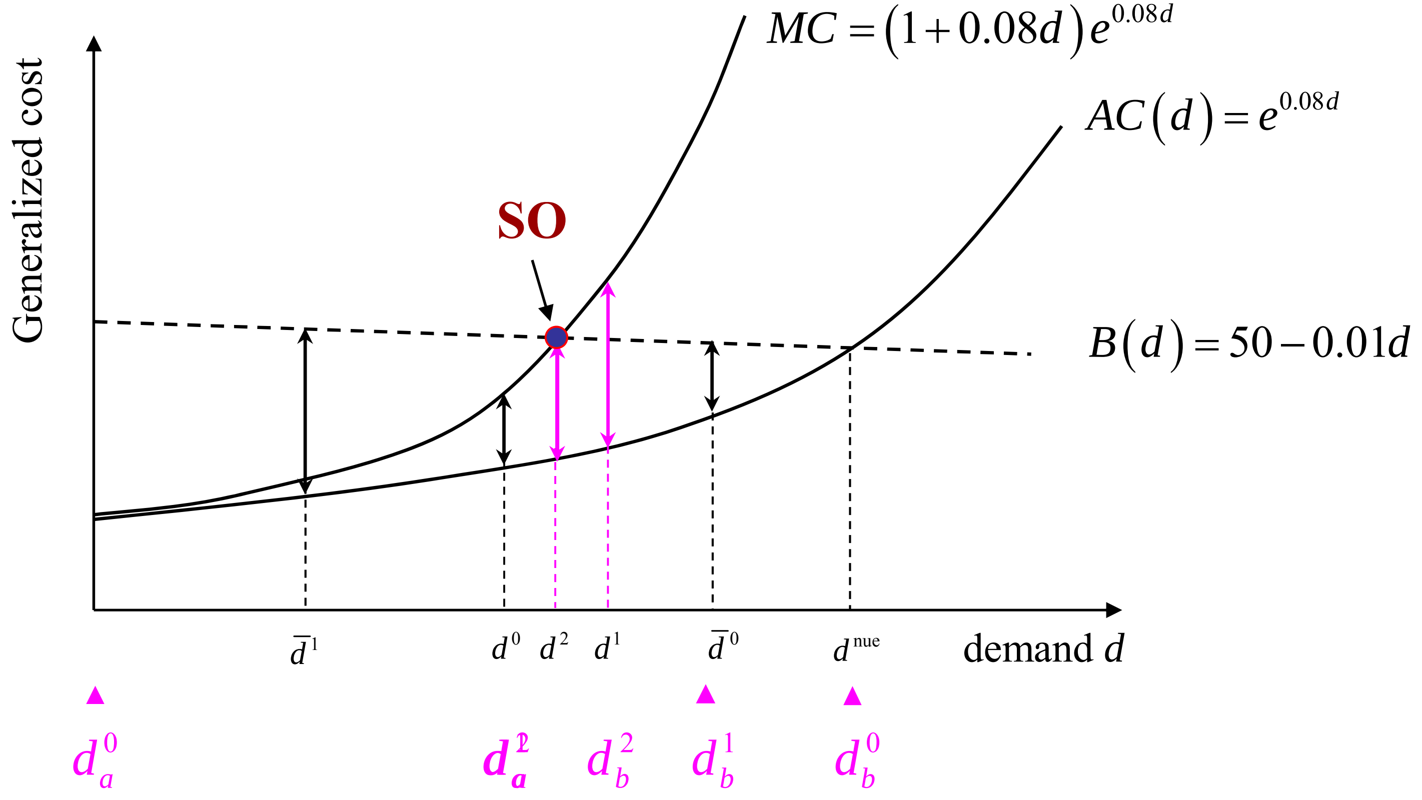
$$\left|d^{(n+1)} - \bar{d}^{(n+1)}\right| < \frac{\left|d^{(n)} - \bar{d}^{(n)}\right|}{2} \quad ?$$



$$\left|d^{(2)} - \bar{d}^{(2)}\right| > \left|d^{(1)} - \bar{d}^{(1)}\right| > \left|d^{(0)} - \bar{d}^{(0)}\right|$$

$\left|d^{n+1} - d^*\right| < \left|d^n - d^*\right|$ is not always guaranteed by Li's bisection method.

Modification of the Bisection Method



$$d_a^{(n+1)} = \max\left\{d_a^{(n)}, \min\left\{d^{(n)}, \bar{d}^{(n)}\right\}\right\}$$

$$d_b^{(n+1)} = \min\left\{d_b^{(n)}, \max\left\{d^{(n)}, \bar{d}^{(n)}\right\}\right\}$$

$$d^{(n+1)} = \frac{d_a^{(n+1)} + d_b^{(n+1)}}{2}$$

$$\left|d_b^{(n+1)} - d_a^{(n+1)}\right| \leq \frac{1}{2}\left|d_b^{(n)} - d_a^{(n)}\right|$$

Trial-and-error Implementation with Unknown Demand Function: Similarity and Difference between Tradable Credit and Congestion Pricing Schemes

	Congestion pricing:	Tradable credit scheme
Decision variable	▪ Link toll charge	▪ Link credit charge ▪ Total amount of credits issued
Available Information	▪ Observed link flow	▪ Observed link flow ▪ Revealed unit credit price

The revealed credit price becomes available, signalizing the necessity for an upward or downward adjustment of the total amount of credits to be issued in a subsequent implementation period.

Trial-and-error Implementation of Tradable Credit Schemes under Demand-supply Equilibrium for a Single Road

- Equilibrium under each trial credit scheme $\left(K^n, \kappa^n\right)$

K^n: total number of tradable credits to be issued

κ^n: credit charge for using the road each time

- If one chooses

$$\kappa(d) = \frac{1}{\rho} d t'(d); \; K = \kappa d$$

where ρis a preset positive constant, then the market equilibrium credit price p^* at system optimum is:

$$p^* = \rho$$

(One can set ⟩ =1 and hence p^* =1.0 at system optimum)

Assumptions

1. The link travel time function is strongly monotone with modulus $\mu > 0$, that is

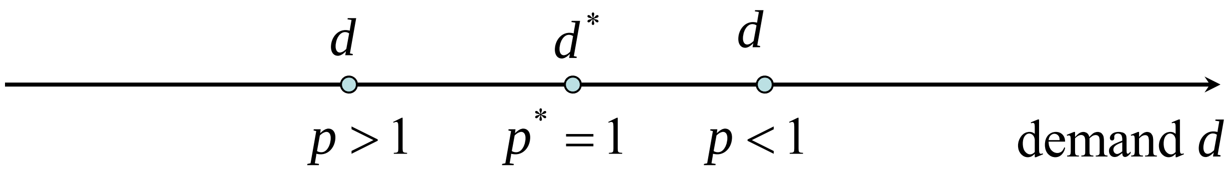
$$\left[t(d) - t(d')\right](d - d') \geq \mu \left|d - d'\right|^2 \quad \forall d, d' \in R$$

2. The benefit function, $B(d)$, and the credit charging function, $\kappa(d) = \frac{1}{\rho} dt'(d)$, are monotone.

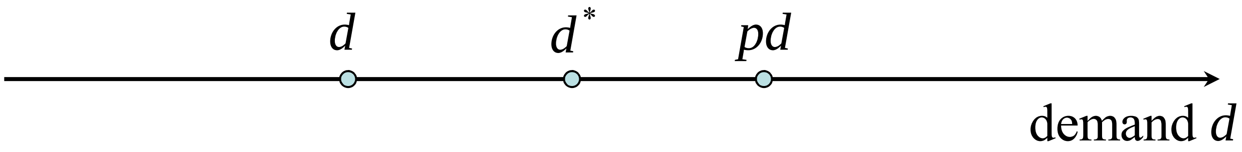
Implication of Observed Credit Price

(Assume 〉=1.0 → p^* =1.0)

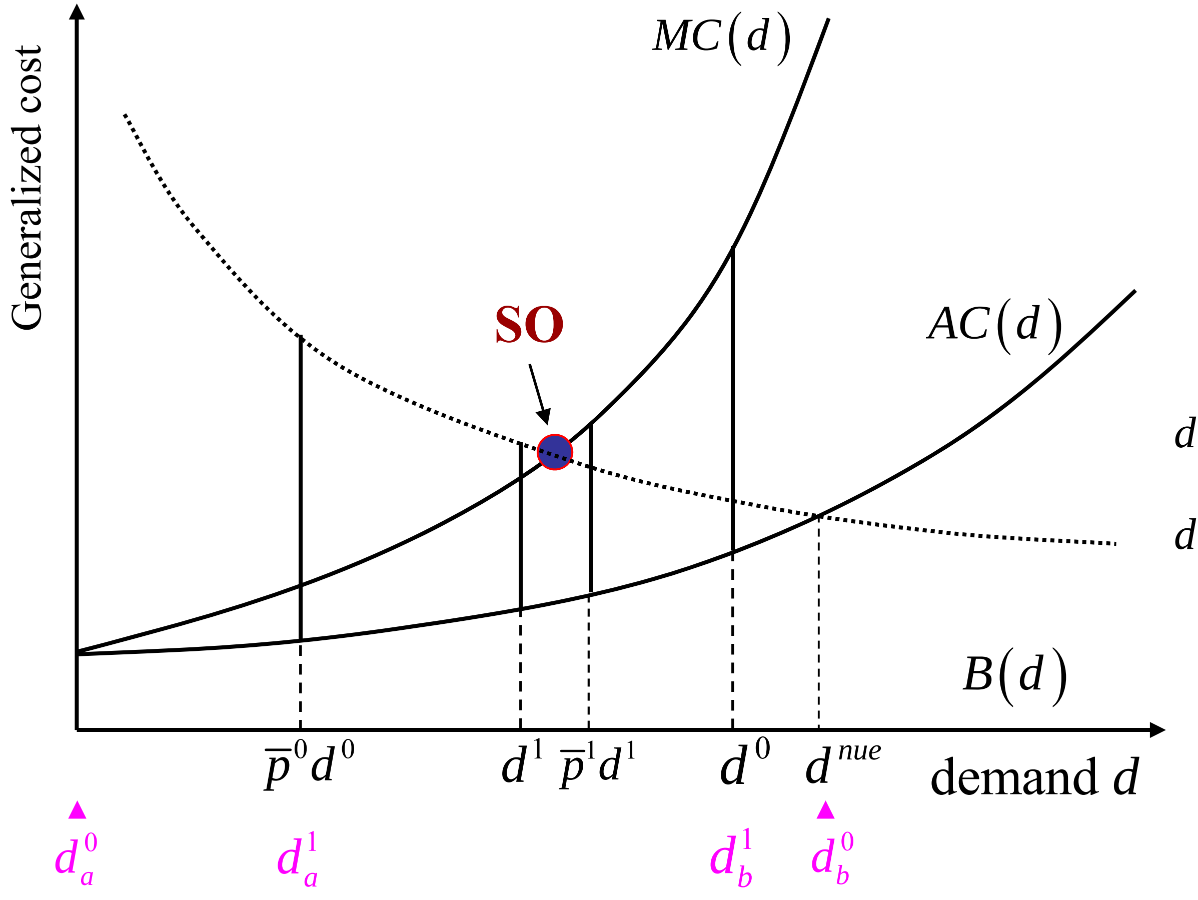
Proposition 1. *For each trial credit scheme with* $\kappa^n = d^n t'\left(d^n\right)$ *and* $K^n = \kappa^n d^n$, $\overline{p}^n = 1 \Leftrightarrow d^n = d^*$; *otherwise,* $\overline{p}^n < 1 \Rightarrow d^n > d^*$ *and* $\overline{p}^n > 1 \Rightarrow d^n < d^*$

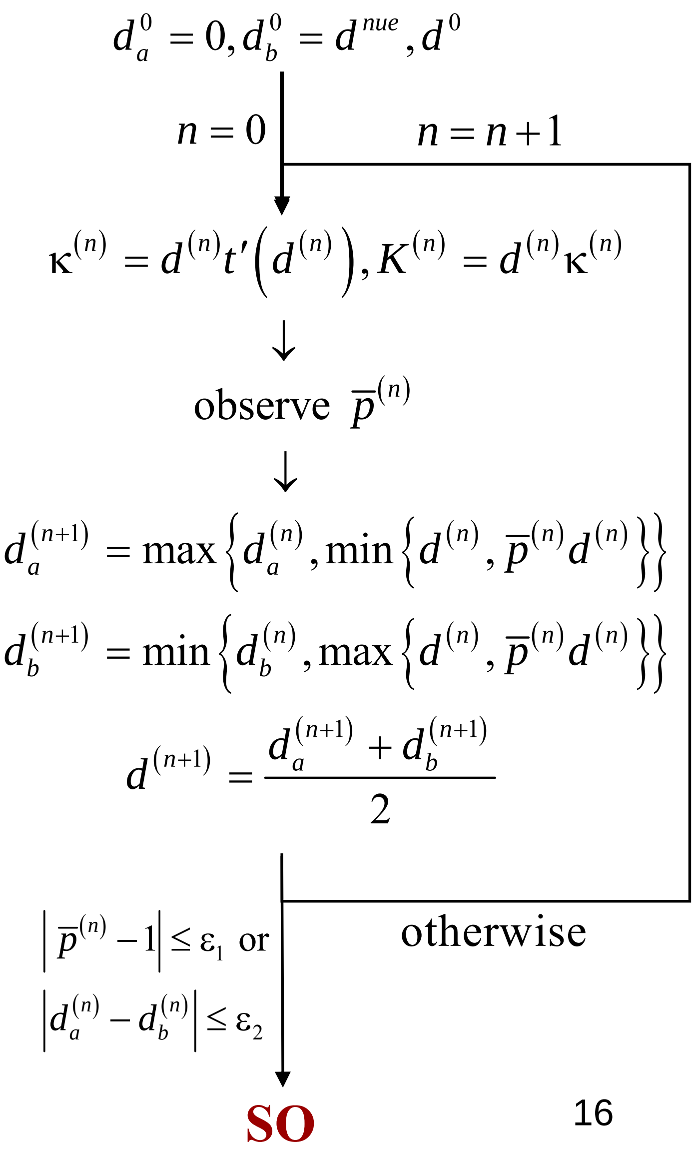


Proposition 2. *For each trial credit scheme with* $\kappa^n = d^n t'\left(d^n\right)$ *and* $K^n = \kappa^n d^n$, *we always have* $\left(\overline{p}^n d^n - d^*\right)\left(d^n - d^*\right) \leq 0$, *and the equality holds only if* $d^n = d^*$.



Trial-and-error Procedure for Tradable Credit Schemes





$$d_a^0 = 0, d_b^0 = d^{nue}, d^0$$

$n = 0$

$n = n + 1$

$$\kappa^{(n)} = d^{(n)} t'\left(d^{(n)}\right), K^{(n)} = d^{(n)} \kappa^{(n)}$$

$\downarrow$

observe $\overline{p}^{(n)}$

$\downarrow$

$$d_a^{(n+1)} = \max\left\{d_a^{(n)}, \min\left\{d^{(n)}, \overline{p}^{(n)} d^{(n)}\right\}\right\}$$

$$d_b^{(n+1)} = \min\left\{d_b^{(n)}, \max\left\{d^{(n)}, \overline{p}^{(n)} d^{(n)}\right\}\right\}$$

$$d^{(n+1)} = \frac{d_a^{(n+1)} + d_b^{(n+1)}}{2}$$

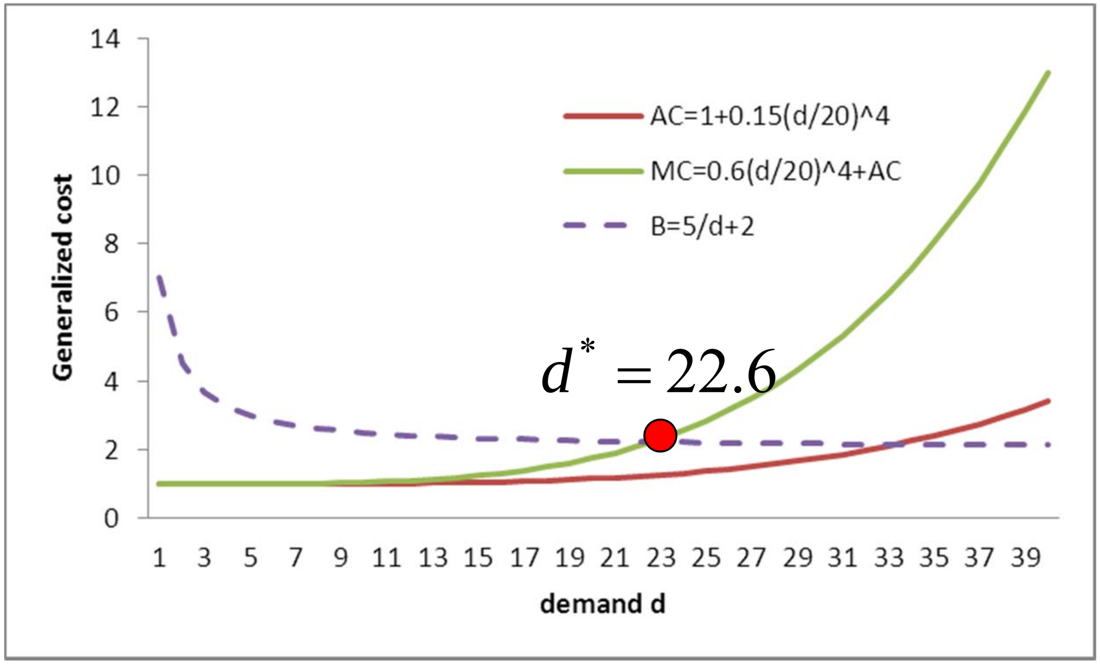
$\left|\overline{p}^{(n)} - 1\right| \leq \varepsilon_1$ or

$\left|d_a^{(n)} - d_b^{(n)}\right| \leq \varepsilon_2$

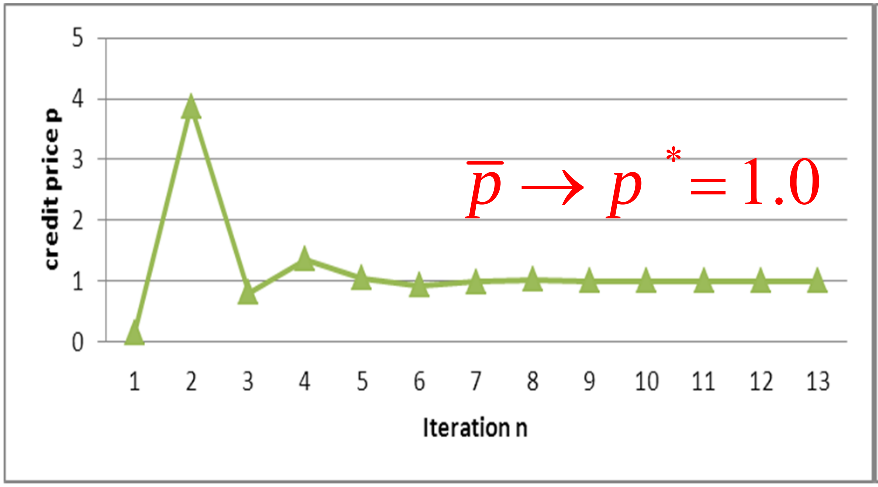
otherwise

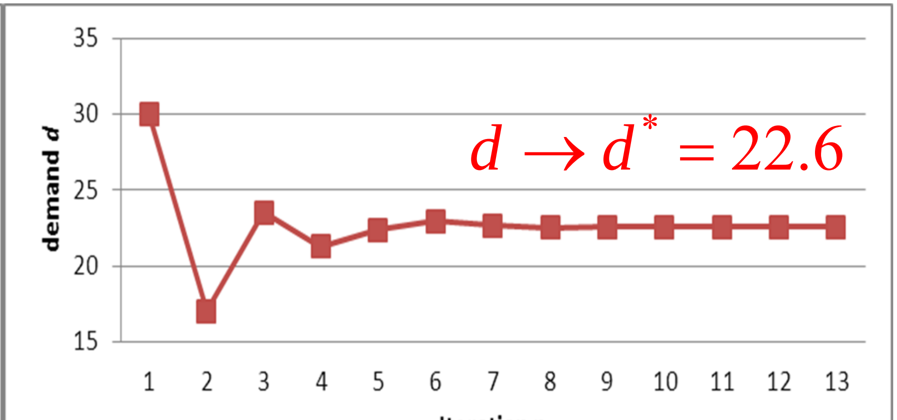
SO

Numerical Example

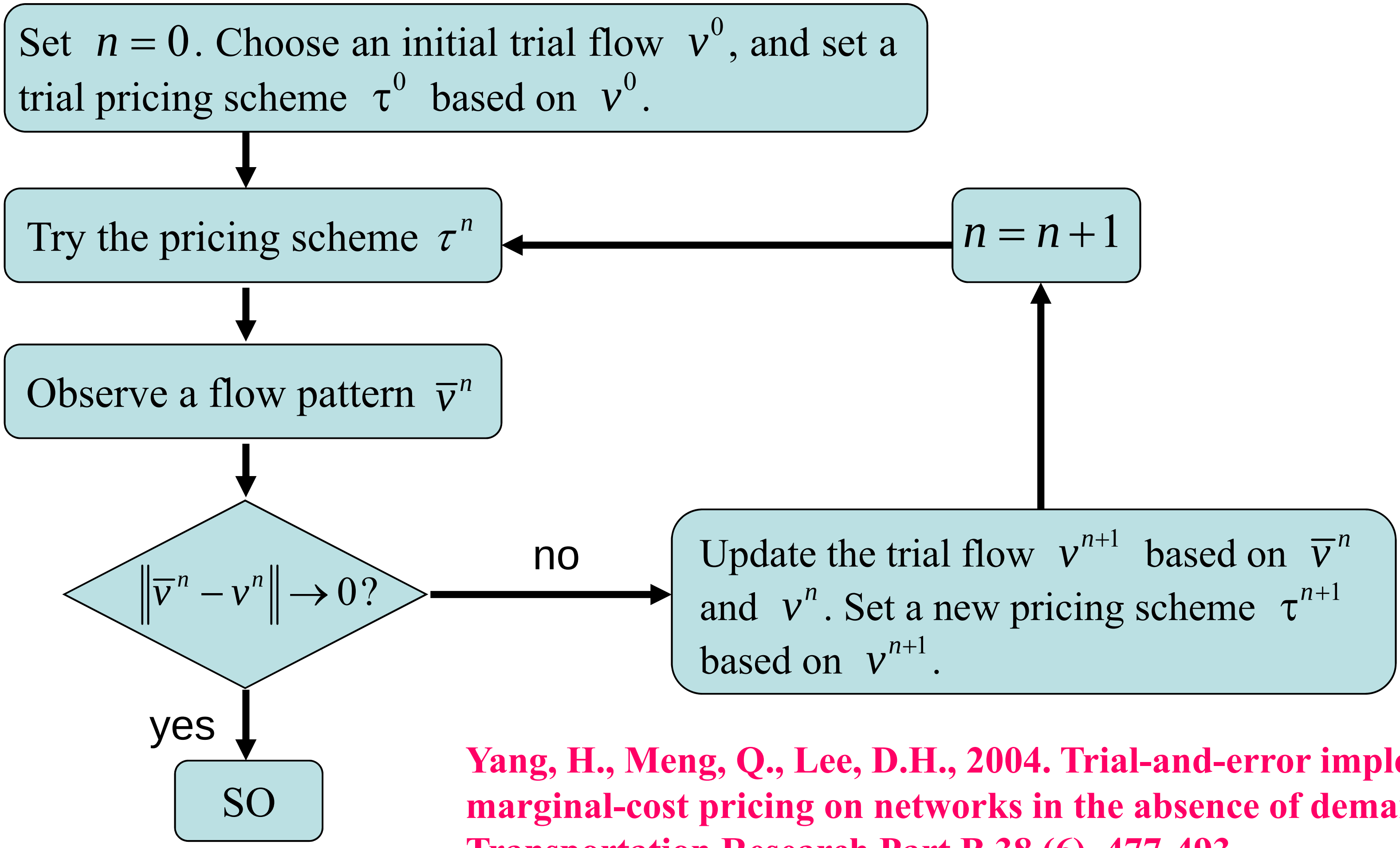


Optimal solution is found after 6 trials





Trial-and-error Implementation of Congestion Pricing on Networks with Unknown Demand Function



Set $n = 0$. Choose an initial trial flow v^0, and set a trial pricing scheme τ^0 based on v^0.

Try the pricing scheme τ^n

Observe a flow pattern $\bar{v}^n$

$\left\|\bar{v}^n - v^n\right\| \to 0?$

no

Update the trial flow v^{n+1} based on $\bar{v}^n$ and v^n. Set a new pricing scheme τ^{n+1} based on v^{n+1}.

$n = n+1$

yes

SO

Yang, H., Meng, Q., Lee, D.H., 2004. Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions. Transportation Research Part B 38 (6), 477-493.

Yang, H., Xu, W., He, B.S. and Meng, Q., 2010. Road pricing for congestion control with unknown demand and cost functions. Transportation Research Part C 18 (2), 157-175.

Extension to General Networks (in Progress)

$$\kappa_a^n = \frac{1}{\rho} v_a^n t_a'\left(v_a^n\right), \quad K^n = \sum_{a \in A} \kappa_a^n v_a^n$$

$$\Downarrow$$

$$\bar{v}^n, \bar{p}^n$$

$$\Downarrow$$

$$v^{n+1} = \max\left\{v^n - \alpha^n h\left(v^n, \bar{v}^n, \bar{p}^n\right), 0\right\}$$

where $h\left(v^n, \bar{v}^n, \bar{p}^n\right)$ is a 'residual' function

Han, D. and Yang, H. (2009) Congestion pricing in the absence of demand functions. *Transportation Research* 45E, No.1, 159-171.

Other Related Ongoing Research:

- *Tradable credit scheme with heterogeneous travelers (discrete set of VOT or continuously distributed VOT) (Wang et al., 2011; Zhu et al.. 2011);*
- *Tradable credit scheme with transaction costs (Nie, 2011)*
- *Tradable credit scheme with income effects (Yin et al., 2011)*
- *Managing bottleneck congestion with tradable credits (Xiao and Zhang, 2010)*
- *Tradable credit schemes for multi-objective optimization (travel time versus emissions) (Chen and Yang, 2011)*
- *Stochastic user equilibrium and tradable credit scheme (Sumalee et al., 2011)*
- *Evolution of price and flow under a tradable credit scheme (Ye and Yang, 2011)*