

The Effects of Competition on Air Traffic Management Initiatives

Amy Kim & Mark Hansen
UC Berkeley

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Ground transportation system

- Users are making choices to minimize their individual travel costs
- Not system-optimal

Air traffic flow management (ATFM)

- Centrally controlled
- However, airlines also attempt to minimize their travel costs through whatever means possible

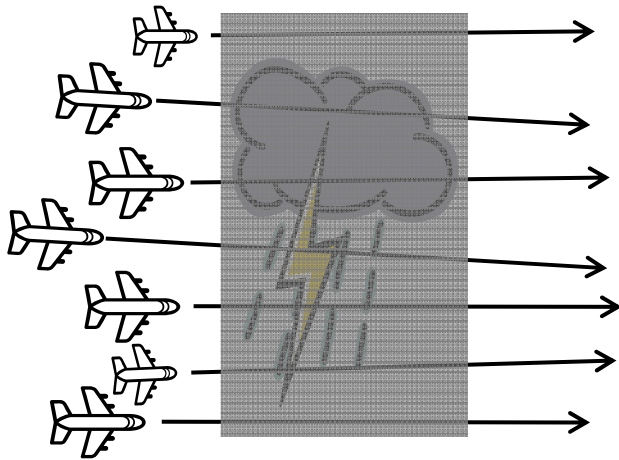
Can a simple traffic assignment analysis be used to gain insights into airline behavior?

Outline

- Background & motivation
- Modelling framework
 - *Network*
 - *Flight cost model*
 - *Allocation schemes*
- Competitive behavior
- Ongoing/future work

Airspace Flow Program (AFP)

Present Concept



Flight ID	ORIG	DEST	Scheduled Departure Time	AFP Departure Time
AWE161	SAN	PHL	13:30	13:53
DAL72	SFO	JFK	13:30	13:58
AAL1678	DFW	BWI	13:45	14:15
UAL916	SEA	IAD	13:55	15:05
:	:	:	:	:

- 1) Identifies en route areas with severe weather or high demands
- 2) Identifies the flights that are scheduled to fly through it
- 3) Assigns flights **delayed departure times** to meter demand (RBS)
- 4) Allows airlines to file alternate route out of AFP (or cancel flight) if new departure time is undesirable

Airspace Flow Program (AFP)

Future Concept

Flight ID	ORIG	DEST	Scheduled Departure Time	AFP Departure Time	ROUTE
AWE161	SAN	PHL	13:30	13:40	...SAV.J207.RDU...
DAL72	SFO	JFK	13:30	13:45	...FLL.ZAPPA1.ZAPPA...
AAL1678	DFW	BWI	13:45	13:45	...J40.RIC.RAVNN2...
UAL916	SEA	IAD	13:55	14:15	...DAWGS4.SPA.J14...
:	:	:	:	:	

- 1) Identifies areas with severe weather or high demands
- 2) Identifies the flights that are scheduled to fly through it
- 3) Assigns flights **delayed departure times AND reroutes** to meter demand

→ *Consider users' preferences*

→ *Allocate resources in way that is acceptable to airlines*

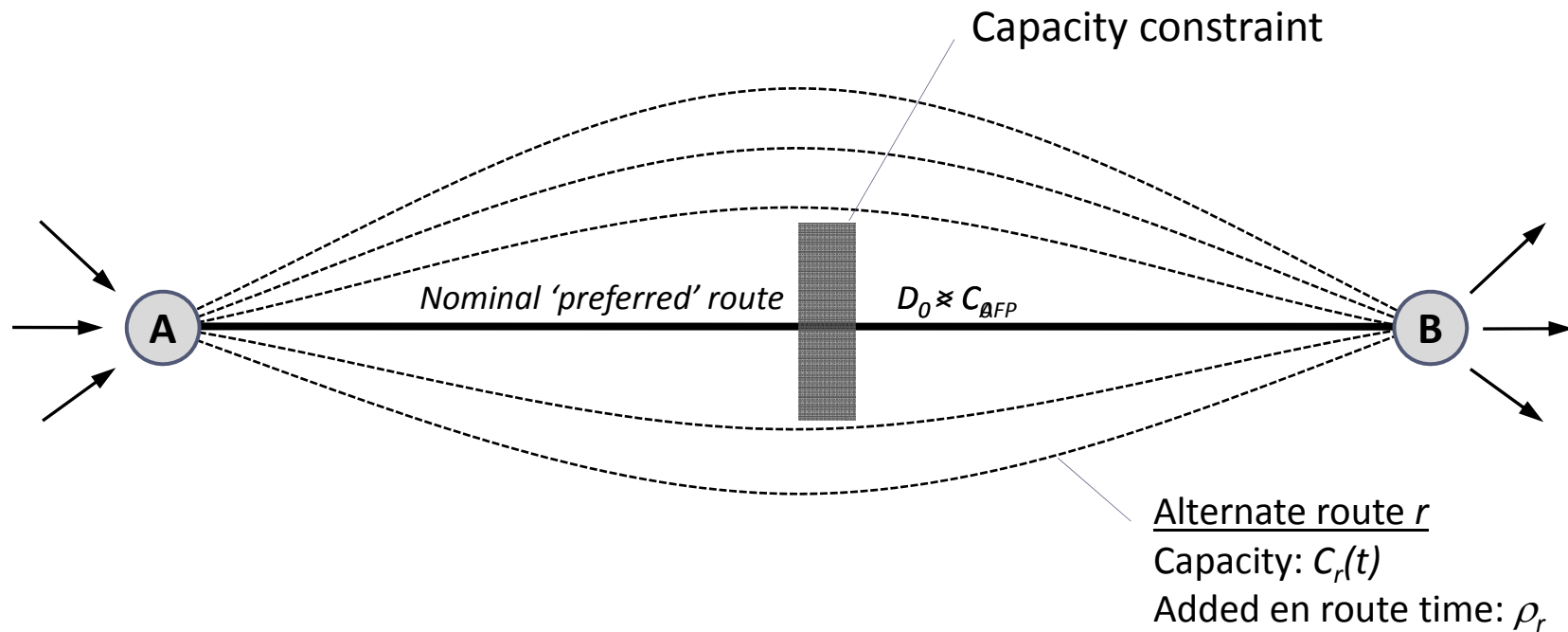
Research Question

- ***If structured airline preferences are considered in airspace resource allocation, how might airlines act to maximize their own benefits?***

Objectives

- Construct **flight assignment schemes** that incorporate different **user input** and **resource allocation rules**
- Understand how user competitive behaviors are manifest in these schemes

Simple network



- Each alternative route has known travel time and capacity
- Aircraft wait on ground until assigned departure “slot” on assigned route

Which route? Which slot?

(Generalized) user flight cost model

cost = en route cost + ground delay cost + other

$$C_{n,r,i} = \alpha_n \cdot \rho_r + d_{r,i} - g_{0,n} + \varepsilon_{n,r}$$

Flight n 's cost to fly route
 r , departing in slot i
(ground delay minutes)

(Generalized) user flight cost model

cost = en route cost + ground delay cost + other

$$C_{n,r,i} = \alpha_n \cdot \rho_r + d_{r,i} - g_{0,n} + \varepsilon_{n,r}$$

Flight n 's en route time to
ground delay ratio
(*ground delay min/airborne min*)

Difference in en route
time between nominal
route and route r ($\rho_r \geq 0$)
(*airborne minutes*)

(Generalized) user flight cost model

cost = en route cost + ground delay cost + other

$$C_{n,r,i} = \alpha_n \cdot \rho_r + d_{r,i} - g_{0,n} + \varepsilon_{n,r}$$

AFP-controlled departure
time on route r , slot i
(ground delay minutes)


Flight n 's original
scheduled departure time
(ground delay minutes)

(Generalized) user flight cost model

cost = en route cost + ground delay cost + other

$$C_{n,r,i} = \alpha_n \cdot \rho_r + d_{r,i} - g_{0,n} + \varepsilon_{n,r}$$

Flight n 's private preferences
regarding route r



Traffic Assignment Schemes

Scheme	User input	Allocation Mechanism
<i>Baseline</i>		
<u>FISO</u> <i>(full information, system optimal)</i>	Route specific cost information (en route cost, other prefs)	Total flight cost minimization
<i>Lower-value user information, more efficient allocation</i>		
<u>PASO</u> <i>(Parametric, system optimal)</i>	General flight cost parameter α_n (air to ground cost ratio)	Total flight cost minimization (with incomplete information)
<i>Higher-value user information, less efficient allocation</i>		
<u>FSFA</u> <i>(first-submitted, first-assigned)</i>	Route specific cost information (en route cost, other prefs)	Incremental flight cost minimization (greedy), with clear fairness rule

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Competitive Behaviour

- Desirable aviation resources are often highly constrained, resulting in a need for ATFM
- Gaming behaviour is a major concern of, and expected in, ATFM

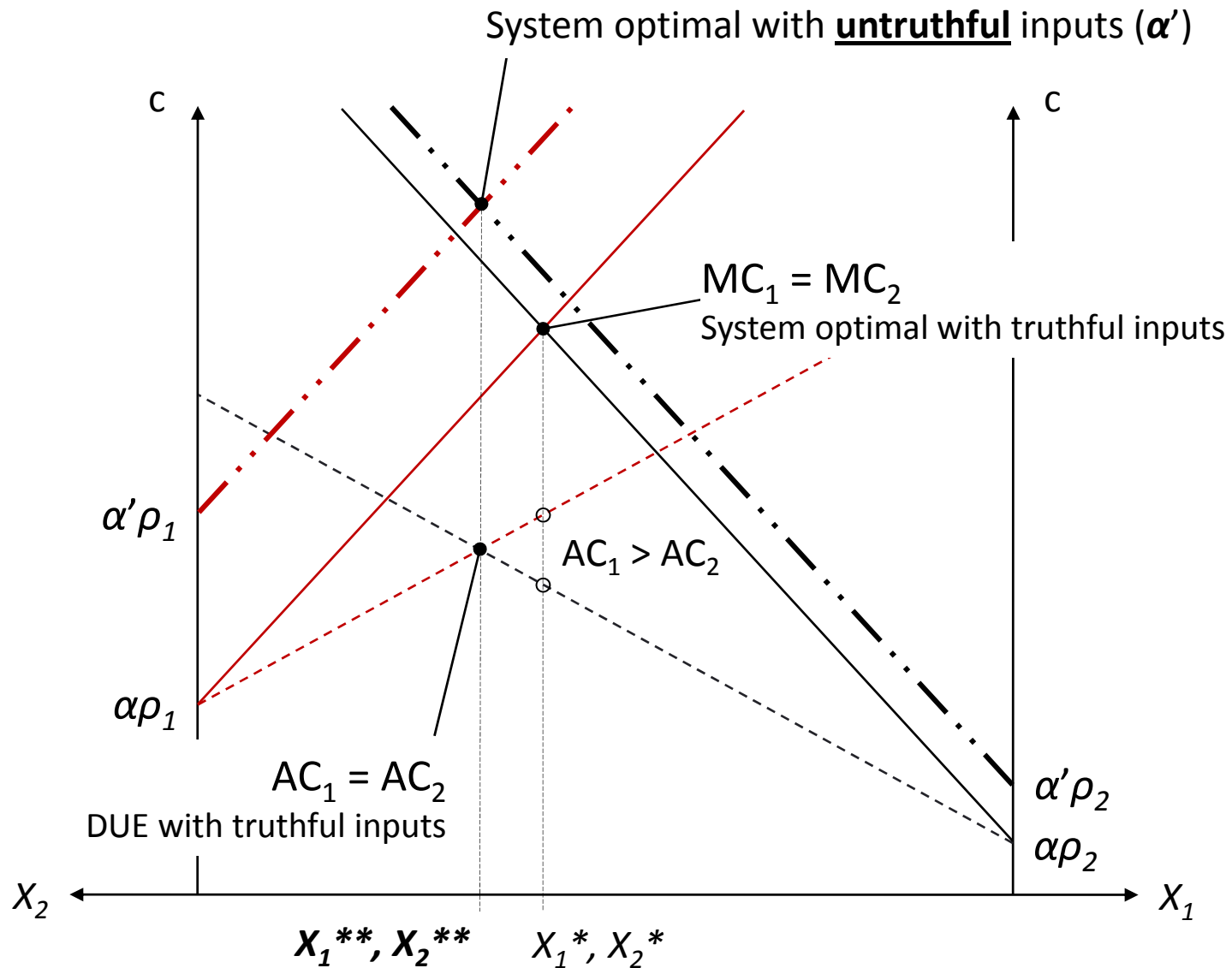
Example: one airline admits to over-scheduling routes through airspace anticipated to be problematic

- Airlines are unlikely to offer “truthful” preference/utility inputs if advantage can be gained by doing otherwise

System-optimal (FISO, PASO)

- Competitive behavior manifest through input values submitted
- Analysis through classic traffic assignment
- Consider a very simple illustration:
 - Two routes, where Route 1 has a longer travel time than Route 2 ($\rho_1 > \rho_2$)
 - Homogeneous flights ($\alpha_n = \alpha$, $\varepsilon_{n,r} = 0$, $g_{o,n} = g_o \forall n$)
 - *Users influence allocation to routes through submitted α ; users have no a-priori information about slot assignment*

System-optimal models



System-optimal models

- In a simple 2 (or 3) route network, with homogeneous α and DUE, competition results in submission

$$\alpha' = 2\alpha$$

System-optimal models

- In a simple 2-route network, with homogeneous α and Gumbel distributed stochastic term, competition results in submission

$$\alpha' = f(\alpha, \vec{\rho}_r, \vec{g}_r, N, \nu)$$

$$\exp\left(\frac{1}{\nu}\left(\alpha - \frac{1}{2}\alpha'\right)(\rho_1 - \rho_2)\right) = \frac{Ng_1 - \alpha'(\rho_2 - \rho_1)}{Ng_2 + \alpha'(\rho_2 - \rho_1)}$$

Results: system-optimal model

- System-optimality may be an impractical goal with structured preference inputs
- Competing airlines will misrepresent their cost model inputs such that assignments move towards user equilibrium

- By assuming truthful preference inputs for FISO:
Comparing a system-optimal (FISO) model against FSFA is meaningless
- By assuming untruthful preference inputs in FISO:
Valid comparison between system-optimal model (FISO) and FSFA

FSFA

- No incentive to misrepresent input information: competitive behavior manifest through submission TIME
- Allocation depends on place in the submissions order

*Start of
planning period*

*End of planning period,
beginning of AFP*



Submit EARLIER

Maximize likelihood of obtaining desirable resource before competitor does

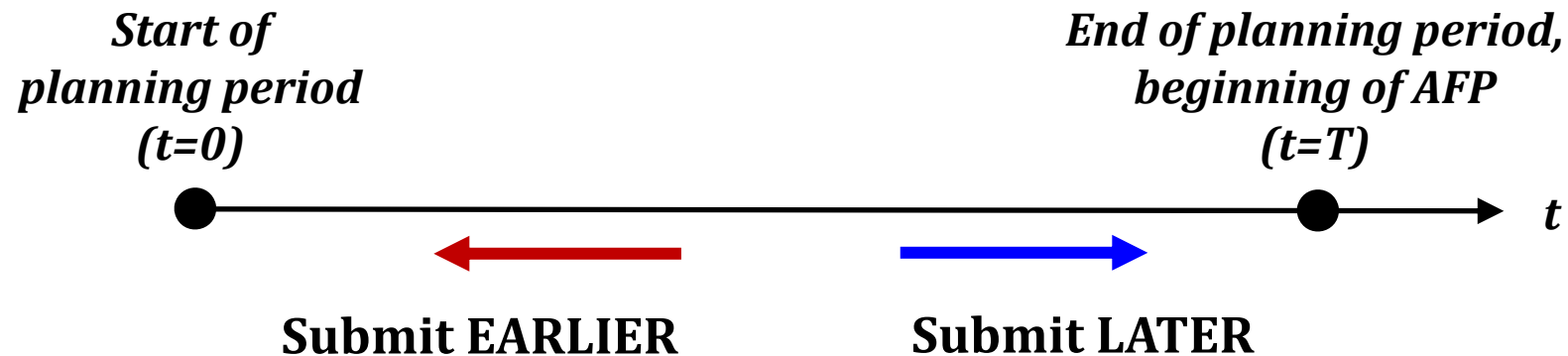
Submit LATER

Decreasing cost of uncertainty regarding internal operations and external conditions

FSFA (cont'd)

- Gain insights about airline behavior through a competition model
- Symmetric game
- Rational players
- Incomplete information game
 - Each player is of a (privately-known) type
 - Type indicates intra-airline uncertainty level during submission process
 - Players have general beliefs about other players' types (distribution over population)

FSFA Payoff Function



$$\max E[\pi(t)] =$$

Maximize

Minimize

$$E[U(1^{\text{st}})] \cdot P(1^{\text{st}}) + E[U(2^{\text{nd}})] \cdot P(2^{\text{nd}}) + \dots + E[U((N-1)^{\text{th}})] \cdot P((N-1)^{\text{th}}) - S(t)$$

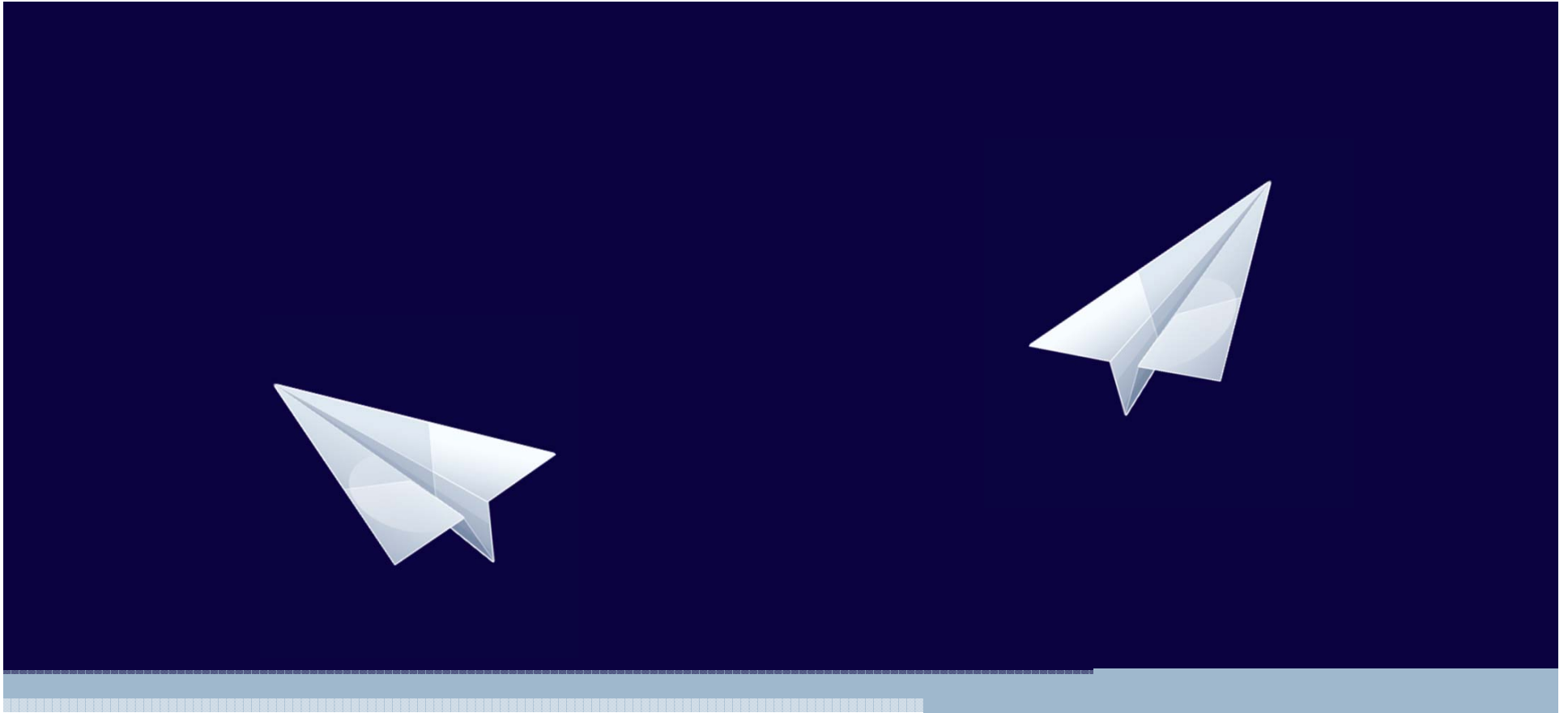
- Equilibrium strategy exists but must be solved numerically
- Results indicates competition drives early submissions

Conclusions

- Classic traffic equilibrium principles are helpful in understanding airline preference input submission behavior
- System-optimal models: clear incentive to lie
- FSFA: competition encourages early submission
 - *Results suggest that because submission times depend on uncertainty level, they are likely to be dependent on scheduled departure times*

On-going & future work

- In order to perform a comparison of the results of the allocation schemes, change/extend analysis methodologies
 - Heterogeneous flights ($\alpha, g_{0n}, \varepsilon$)
 - Scheduled departure time constraint
- Acknowledge flight ownership by airlines
- Consideration of learned behavior
- Flight cancelations: elastic demand



Thank you!

amy_kim@berkeley.edu

Numerical illustration - PASO vs. FSFA



- Assumptions:
 - PASO is at SUE
 - Flight operators submit inputs in random order in FSFA
- Recursive, quasi-analytic procedure to find FSFA result
 - Employs expected received utility concept of logit discrete choice
 - Assume submissions are received in random order

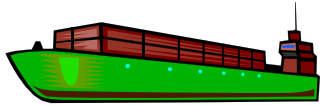


- *Demand*

- Total AFP flights: **$N = 50$**
- Air to ground cost ratio (*ground delay min/en route min*): $\alpha_n \in (1.5, 3.5]$

- *Supply*

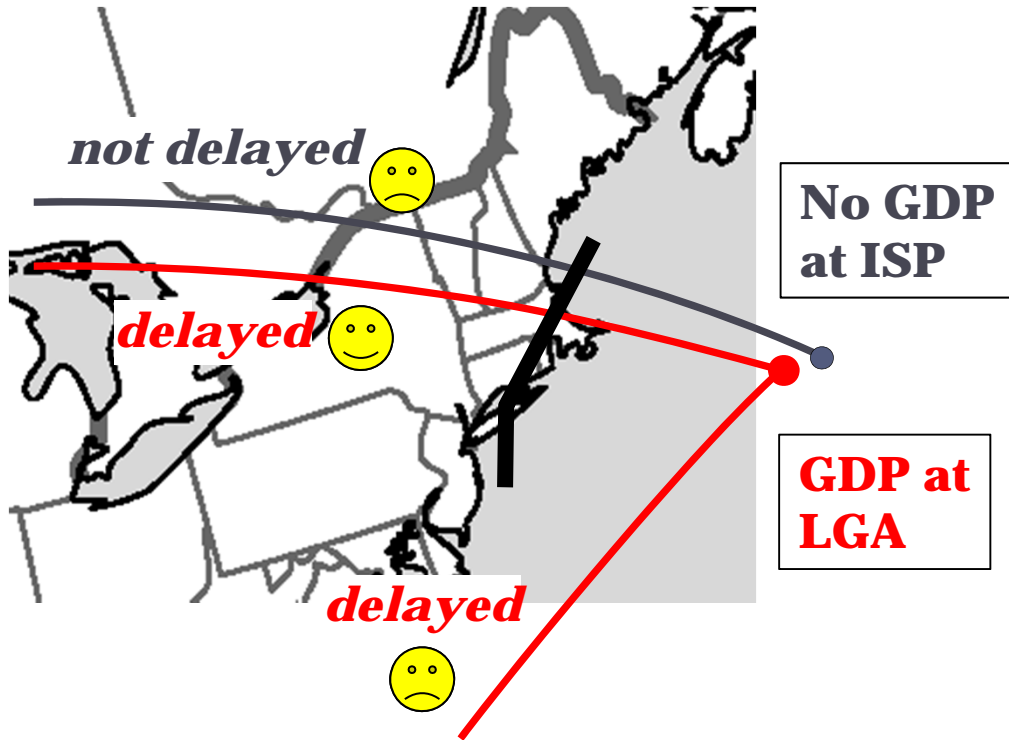
Route	Capacity, C_r (flights/hour)	Departure Headway, g_r (min)	En Route Time (min)	Additional Air Time, ρ_r (min)
1	20	3	130	30
2	10	6	120	20
3	12	5	115	15
4 (nominal)	7.5	8	100	0



How might traffic managers most efficiently and fairly serve different & competing customers when they are uncertain about what their customers want?



En route capacity constraints

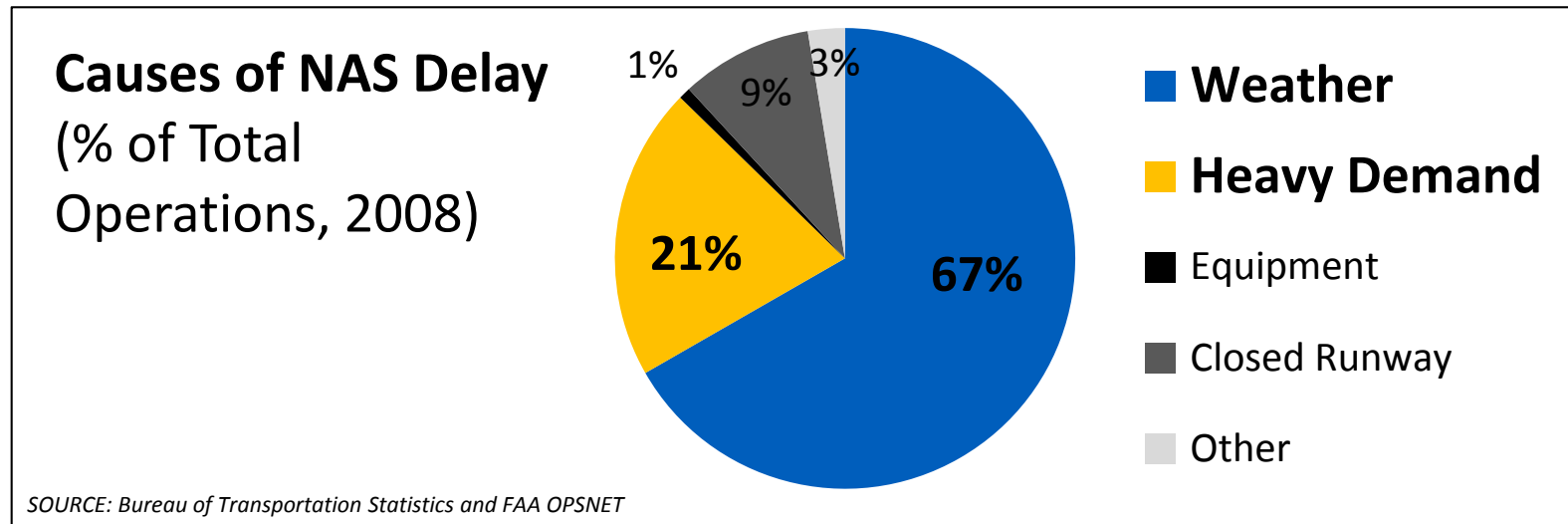


- Until 2005, Ground Delay Program (GDP) used to address en route constraints
- GDPs often delayed flights not directly affected by constraint

- In 2006 the Airspace Flow Program (AFP) replaced GDPs to manage en route constraints

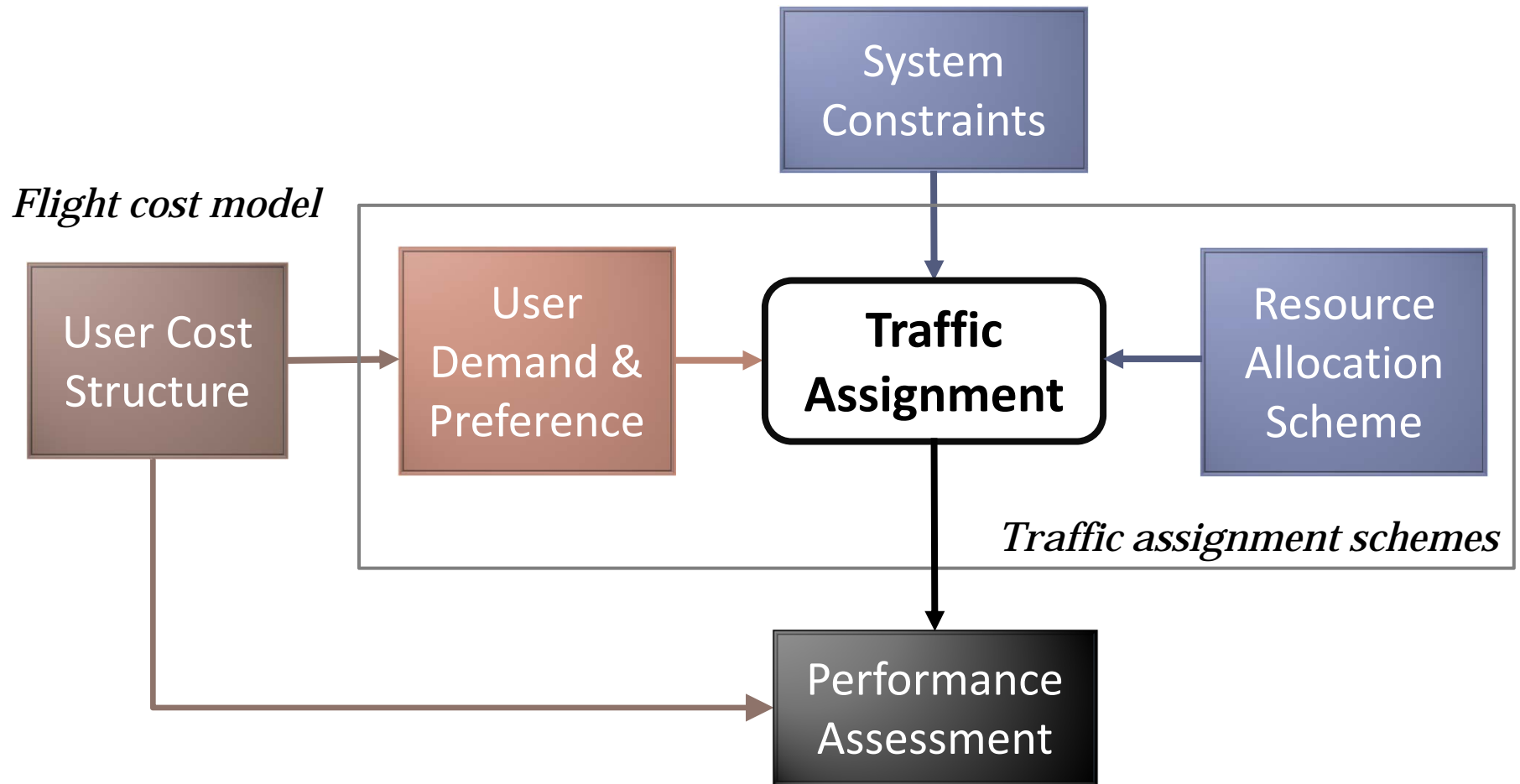
Background

- Air traffic congestion occurs daily in the U.S. airspace system



- As demands grow, delay caused by demand/capacity imbalances also worsen under current practices
- Necessary to more efficiently utilize all available capacity during severe weather and heavy traffic events

➤ ***Improve Traffic Management Programs***



Literature review

- Traffic assignment (DUE, SUE) and game-theoretic counterparts



- Daganzo & Sheffi (1977), Dafermos (1982), Daganzo (1983), etc.
- Rosenthal (1973), Haurie & Marcotte (1985), etc.

- Air traffic flow management (ATFM) models

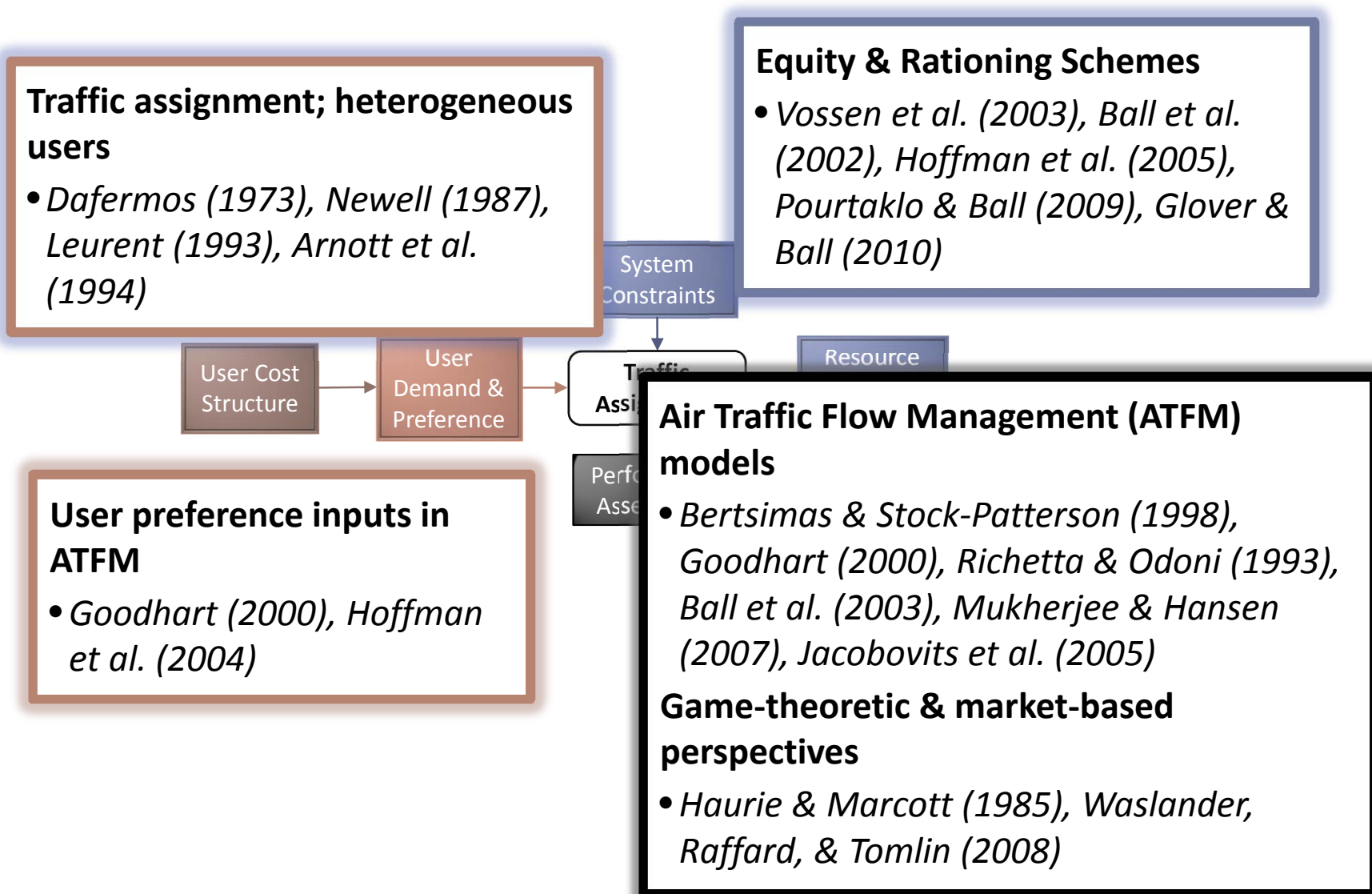


- Bertsimas & Stock-Patterson (1998), Goodhart (2000), Richetta & Odoni (1993), Ball et al. (2003), Mukherjee & Hansen (2007), etc.
- User preference inputs to ATFM
 - Goodhart (2000), Hoffman et al. (2004)
- Equity & resource rationing
 - Vossen et al. (2003), Ball et al. (2002), Hoffman et al. (2005), Pourtaklo & Ball (2009), Glover & Ball (2010)

This research offers...

- A framework to study the impacts of:
 - the FAA's uncertainties about their customers' preferences
 - customers' competitive behaviors (manifest through preference inputs)on resource allocation performance and feasibility.
- A performance assessment from user cost perspective, using random utility theory

Literature review



Total Cost Formulation

$$C = \sum_{n \in F} \alpha_n \rho_{r(n)} + g_{r(n)} i_{r(n)}(n) - g_{0,n} + \varepsilon_{n,r(n)}$$

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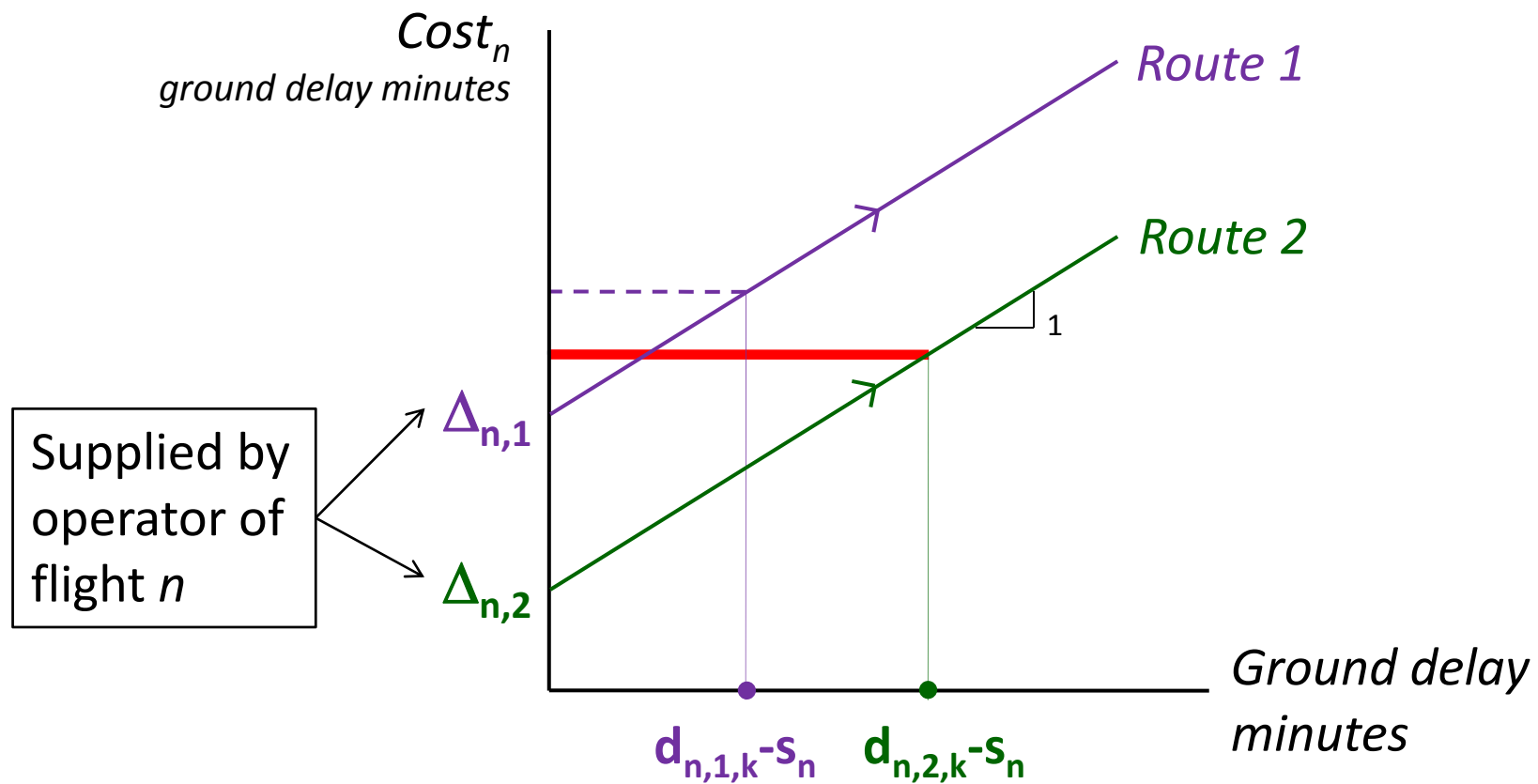
Stated Route Preference (OPT & FSFA)

- Designed to obtain true preferences from airlines, without revealing actual cost information
- Consider that airlines submit (AFP) route specific information about their flights, before ground delay assignments are known

$$c_{n,r,k} = \alpha_n \cdot \rho_r + d_{n,r,k} - s_{0,n} + \varepsilon_{n,r}$$
$$\Delta_{n,r} = \alpha_n \cdot \rho_r + \varepsilon_{n,r}$$

- Allocation decisions are made based on $\Delta_{n,r}$

Stated route preference



$$\Delta_{n,r} = \alpha_n \cdot \rho_r + \varepsilon_{n,r}$$

Model Properties

- Simple example:
 - 2 flights, 2 routes, 1 slot each route
 - Deterministic costs (c_{nr}) of any flight assigned to any route are identical
- *Stochastic terms determine how resources are allocated in each assignment scheme*

$$E[C_{FISO}] = C + E[\min(\varepsilon_{11} + \varepsilon_{22}, \varepsilon_{12} + \varepsilon_{21})]$$

$$E[C_{FSFA}] = \frac{1}{2} \left[\begin{array}{l} C + E[\min(\varepsilon_{11}, \varepsilon_{12}) + \varepsilon_{22}] \quad \text{if } \min(\varepsilon_{11}, \varepsilon_{12}) = \varepsilon_{11} \\ C + E[\min(\varepsilon_{11}, \varepsilon_{12}) + \varepsilon_{21}] \quad \text{if } \min(\varepsilon_{11}, \varepsilon_{12}) = \varepsilon_{12} \\ + \frac{1}{2} \left[\begin{array}{l} C + E[\min(\varepsilon_{21}, \varepsilon_{22}) + \varepsilon_{12}] \quad \text{if } \min(\varepsilon_{21}, \varepsilon_{22}) = \varepsilon_{21} \\ C + E[\min(\varepsilon_{21}, \varepsilon_{22}) + \varepsilon_{11}] \quad \text{if } \min(\varepsilon_{21}, \varepsilon_{22}) = \varepsilon_{22} \end{array} \right] \end{array} \right]$$

$$E[C_{PASO}] = C + E[\varepsilon_{1r}] + E[\varepsilon_{2(-r)}]$$

Model Properties (cont'd)

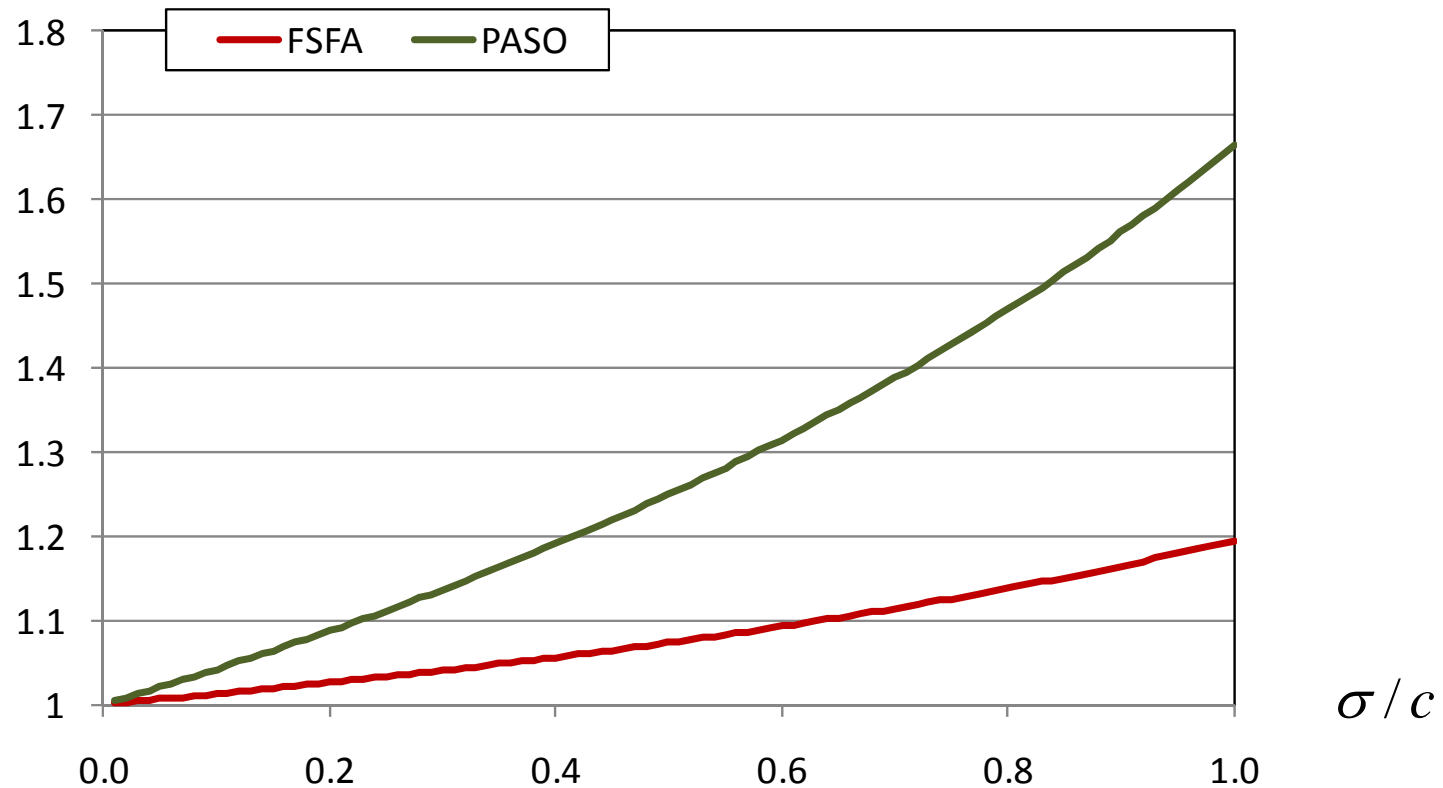
- If $\varepsilon_{nr} \sim N(0, \sigma)$ iid, use results of the maximum of two normal iid variables to find

$$\longrightarrow E[C_{FISO}] = C - \frac{\sqrt{2}\sigma}{\sqrt{\pi}}, E[C_{FSFA}] = C - \frac{\sigma}{\sqrt{\pi}}, E[C_{PASO}] = C$$

- σ represents level of traffic managers' uncertainties about airlines' flight costs
- Compare expected cost of each assignment scheme over increasing values of σ

Model Properties (cont'd)

$$E[C_{FISO}] = C - \frac{\sqrt{2}\sigma}{\sqrt{\pi}}, E[C_{FSFA}] = C - \frac{\sigma}{\sqrt{\pi}}, E[C_{PASO}] = C$$

$$\frac{C_{Model}}{C_{FISO}}$$


Numerical Investigation

- Generate results for more real-world scale examples
- Incorporate heterogeneity, larger flight populations

Numerical investigation

Overview

Flight cost

Assignment

Example

User input

Final Remarks

Methodology

For each value of σ :

1. Randomly draw $\varepsilon_{n,r}$ values from distribution $N(0, \sigma)$.
2. Randomly and independently assign each flight $\alpha, g_\alpha, \varepsilon_{n,r} (\forall r)$.
3. Traffic assignment:

Linear Assignment Problem

FISO: min C

PASO: min \hat{C}

Sequential Assignment

FSFA: By random order

RBS: By original schedule

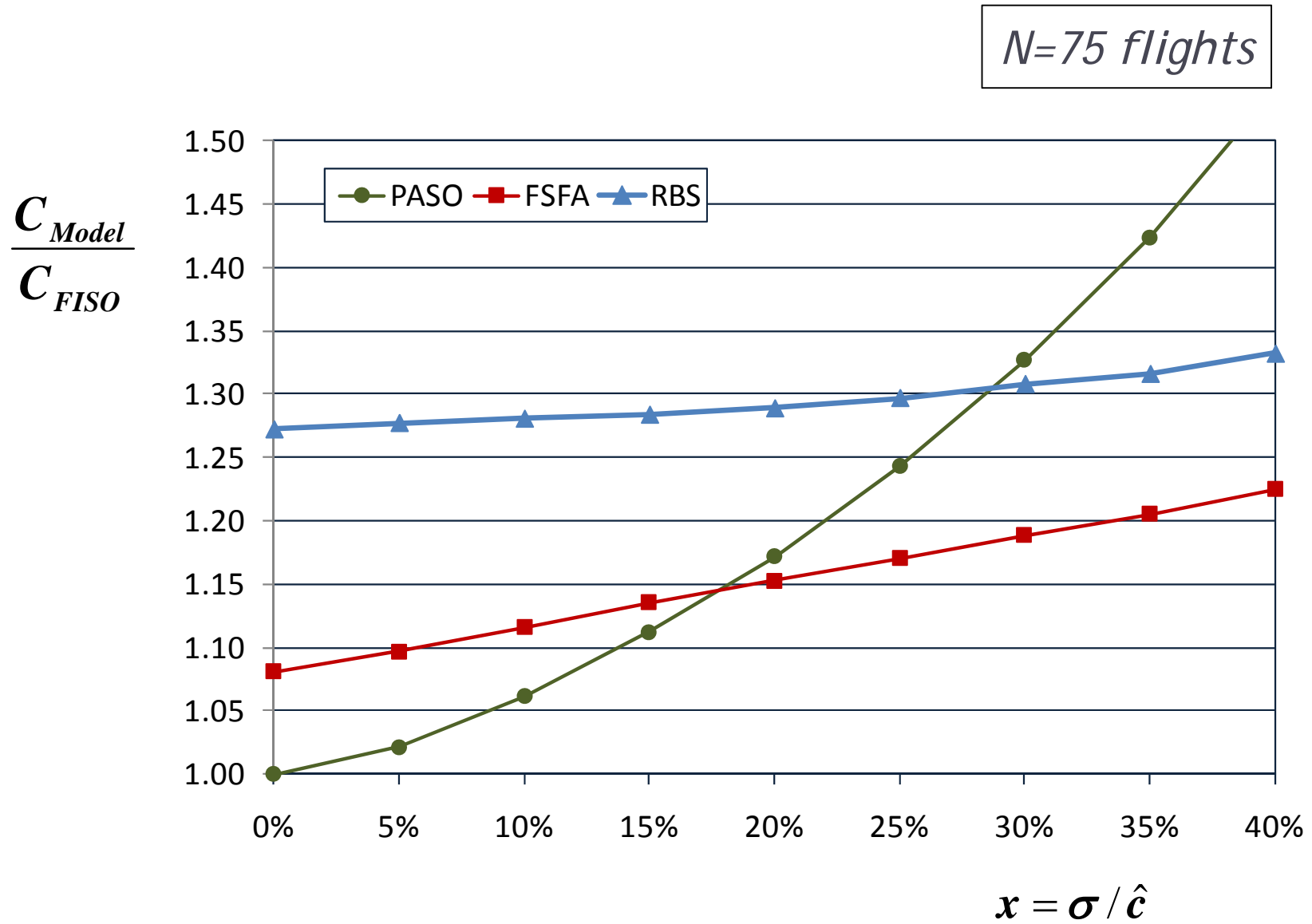
4. Flight cost assessment.
5. Perform 5-10,000 iterations of (1) through (4).

- *Demand*

- Total flights in AFP: **$N = 75$**
- Air to ground cost ratio (*ground delay min/en route min*): **$\alpha_n \in (1.5, 2.5]$**
- Original (pre-AFP) scheduled demand: **$D_0 = 80$ flights/hour**
(or headways **$g_0 = 0.75$ ground delay minutes/flight**)

- *Capacity ($C_{AFP} < D_0$)*

Route	Capacity, C_r (flights/hour)	Departure Headway, g_r (min)	En Route Time (min)	Additional Air Time, ρ_r (min)
1	24	2.5	135	35
2	20	3	130	30
3	10	6	120	20
4	12	5	115	15
5 (nominal)	7.5	8	100	0
$C_{AFP} =$	73.5			



Sensitivity

	C'_{PASO}	C'_{FSFA}	C'_{RBS}
Demand rate increase ($D_0 \uparrow$)	↑	↓	↓
AFP size increase ($N, T \uparrow$)	↑	↑	↑
α_{max} increase	↓	?	?

Where $C'_{PASO} = C_{PASO} / C_{FISO}$

User input: Gaming Behavior

Overview

Flight cost

Assignment

Example

User input

Final Remarks

FSFA

- Introduces gaming behavior in TIME of submission (rather than the information itself)
- Allocation depends on the flight's place in the submissions order

*Start of
planning period*

*End of planning period,
beginning of AFP*



Submit EARLIER

Maximize likelihood of
obtaining desirable resource
before competitor does
(what you think you want)

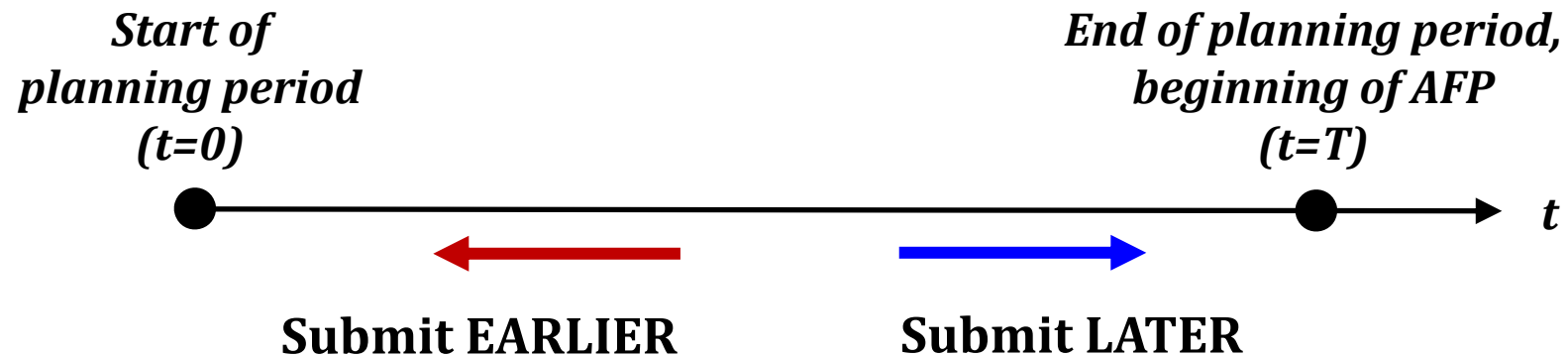
Submit LATER

Decreasing cost of uncertainty
regarding internal operations
and external conditions
(what you actually need)

FSFA (cont'd)

- Gain insights about airline behavior through a competition model
- Symmetric game
- Rational players
- Incomplete information game
 - Each player is of a (privately-known) type
 - Type indicates intra-airline uncertainty level during submission process
 - Players have general beliefs about other players' types

FSFA Payoff Function



$$\max E[\pi(t)] = \overset{\text{Maximize}}{\boxed{E[U(1st)] \cdot P(1st) + E[U(2nd)] \cdot P(2nd)}} - \overset{\text{Minimize}}{\boxed{S(t)}}$$

FSFA Equilibrium Strategy

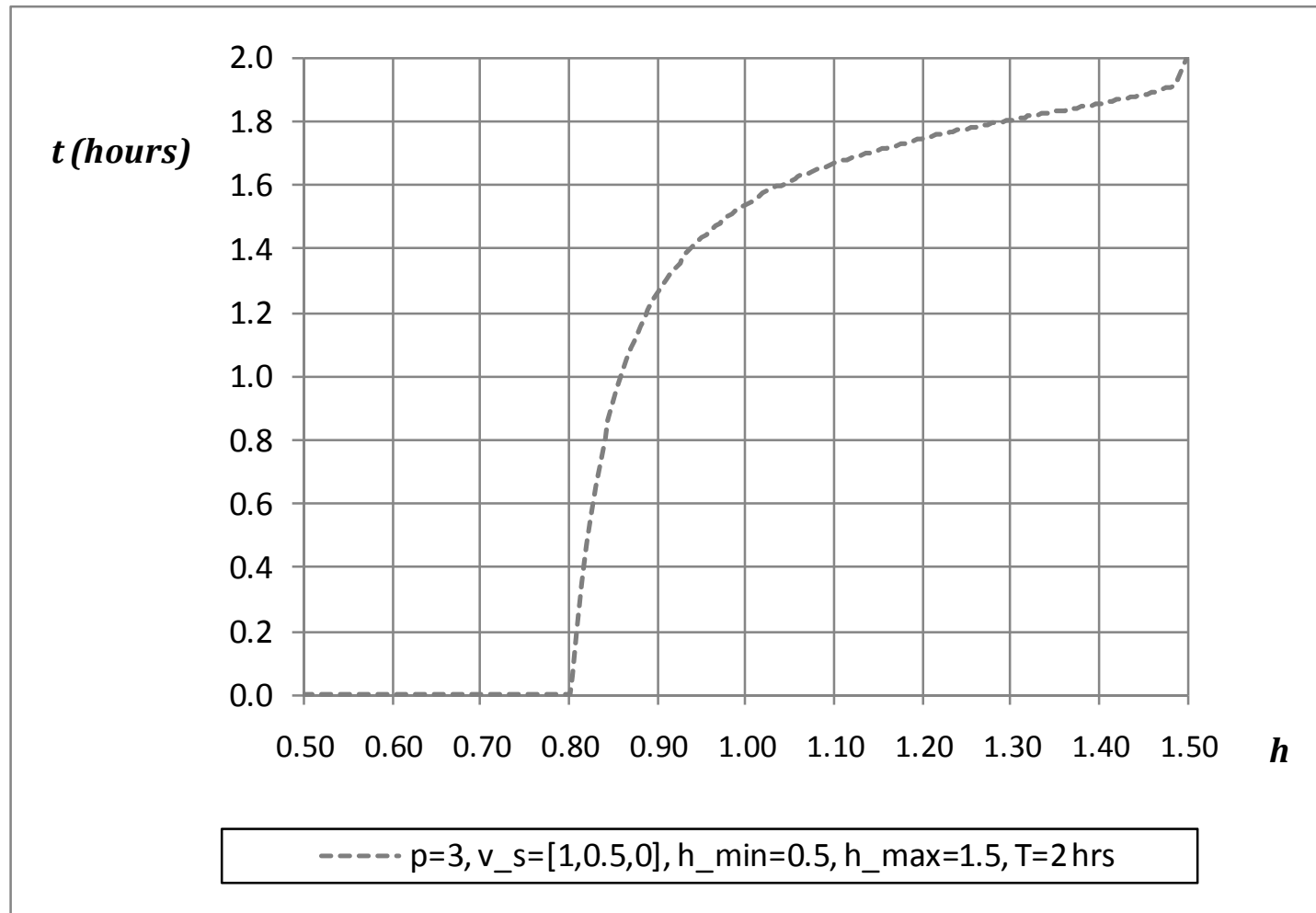
- Player n 's submission time, $t_n = f(\text{type}_n; \text{distn of types, other parameters})$ is solution to:

$$1 - \left(\sum_{s \in S} \exp\left(\frac{v_s T}{(T - t_n)p}\right) \right)^{-1} \cdot \sum_{s \in S} v_s \exp\left(\frac{v_s T}{(T - t_n)p}\right)$$

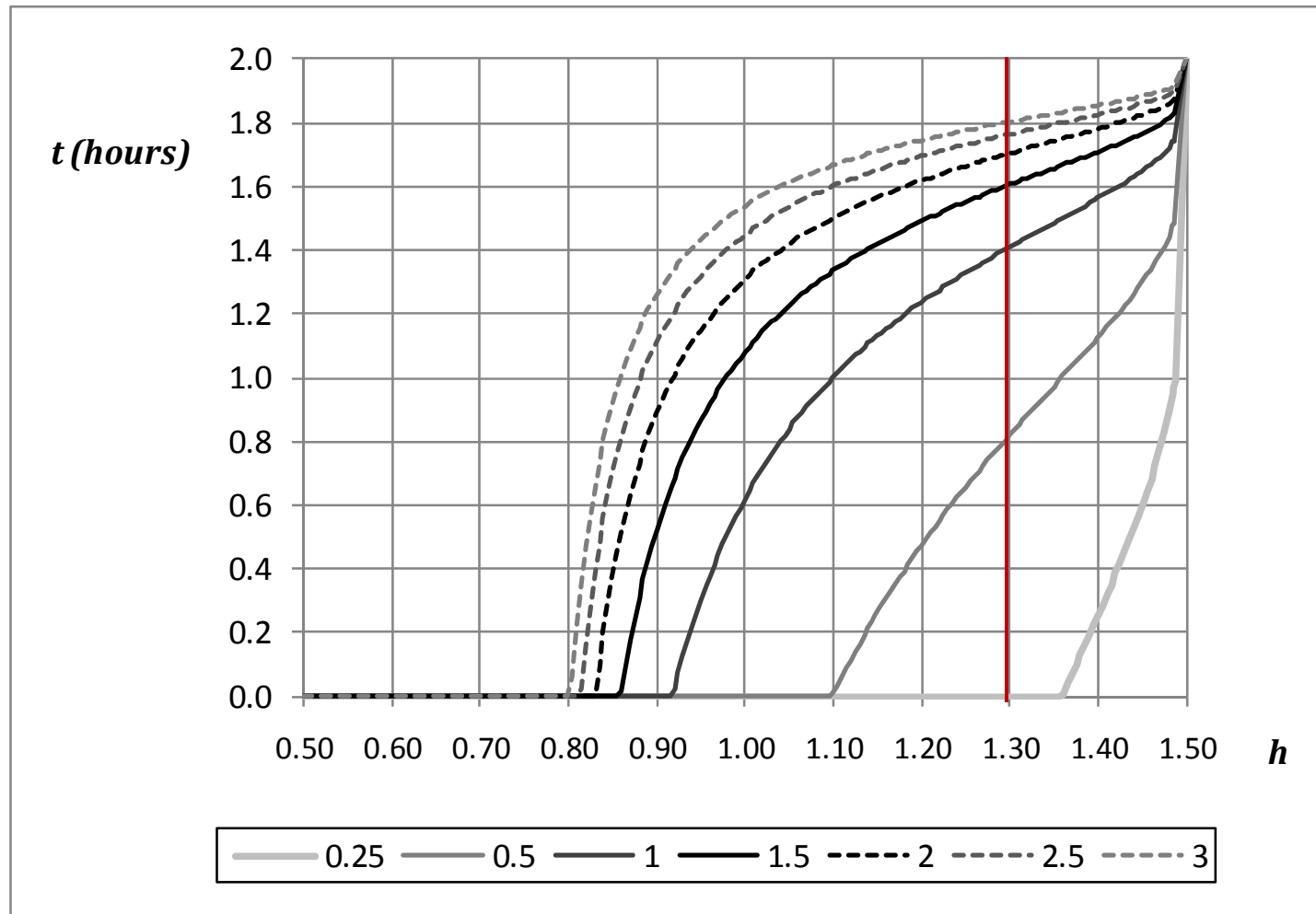
$$= \frac{2[(r_1 h_{max} - r_2(h_{min} + h_{max}))(\ln h_{max} - \ln h_n) + (r_1 - 2r_2)(h_n - h_{max})]}{(h_{max} - h_{min})^2}$$

- Types are distributed $h \sim U(h_{min}, h_{max})$

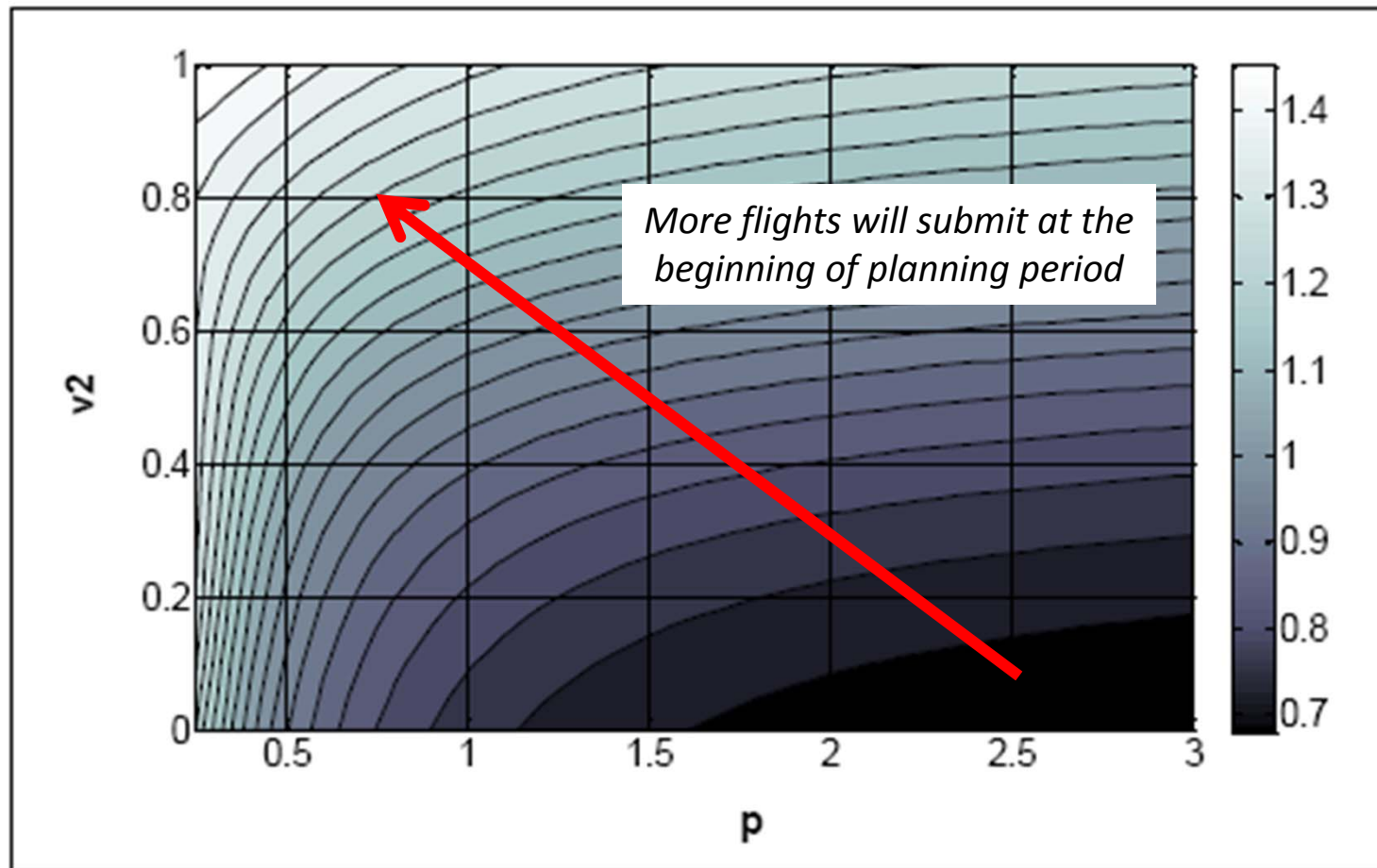
FSFA Equilibrium Strategy Example



FSFA Equilibrium Strategy Example



FSFA Decision Chart (h_0)



Final Remarks

- Insights and implications for program planning & design
- Analysis takes perspective not traditionally used in the ATFM literature
- Competitive behavior is a major concern in airspace capacity allocation programs

Ongoing & Future Work

- Reformulate FSFA competition model to account for differing scheduled departure times ($g_{0,1} \neq g_{0,2} \neq \dots \neq g_{0,N}$)
- Flight groups by airline? Market? Aircraft type?
- Beyond simple networks: probit vs. logit
- Flight cancelations: elastic demand

Model description & mathematical setup

Methodology

- FISO & PASO: linear assignment problem

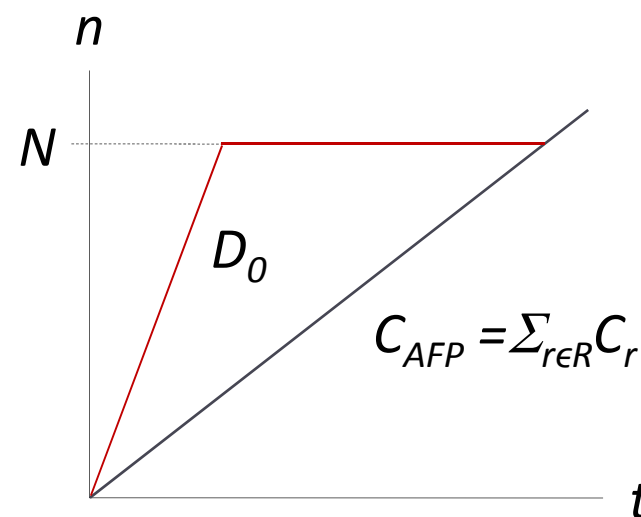
Objective Fn: $\min_{x_{n,j} \forall n,j} C = \sum_{n \in F, j \in J: d_j \geq g_{0,n}} c_{n,j} \cdot x_{n,j}, \quad \epsilon_{n,r} \sim N(0, \sigma)$

Constraints: $d_j \geq g_{0,n}, \sum_{j \in J} x_{n,j} = 1, \forall n, \sum_{n \in F} x_{n,j} \leq 1, \forall j, x_{n,j} \in \{0,1\}, \forall n, j$

- 5,000-10,000 iterations of each model at each σ value

Approximation (non-binding schedule constraint)

- No explicit constraint that Flight n 's AFP departure time must occur prior to n 's original scheduled departure time, or $d_{n,r,k} \geq s_n$
- Reasonable under condition where $D_0 \gg C_{AFP}$
- Restrictive, but makes FSFA and Parametric models analytically tractable



Parametric Approximation

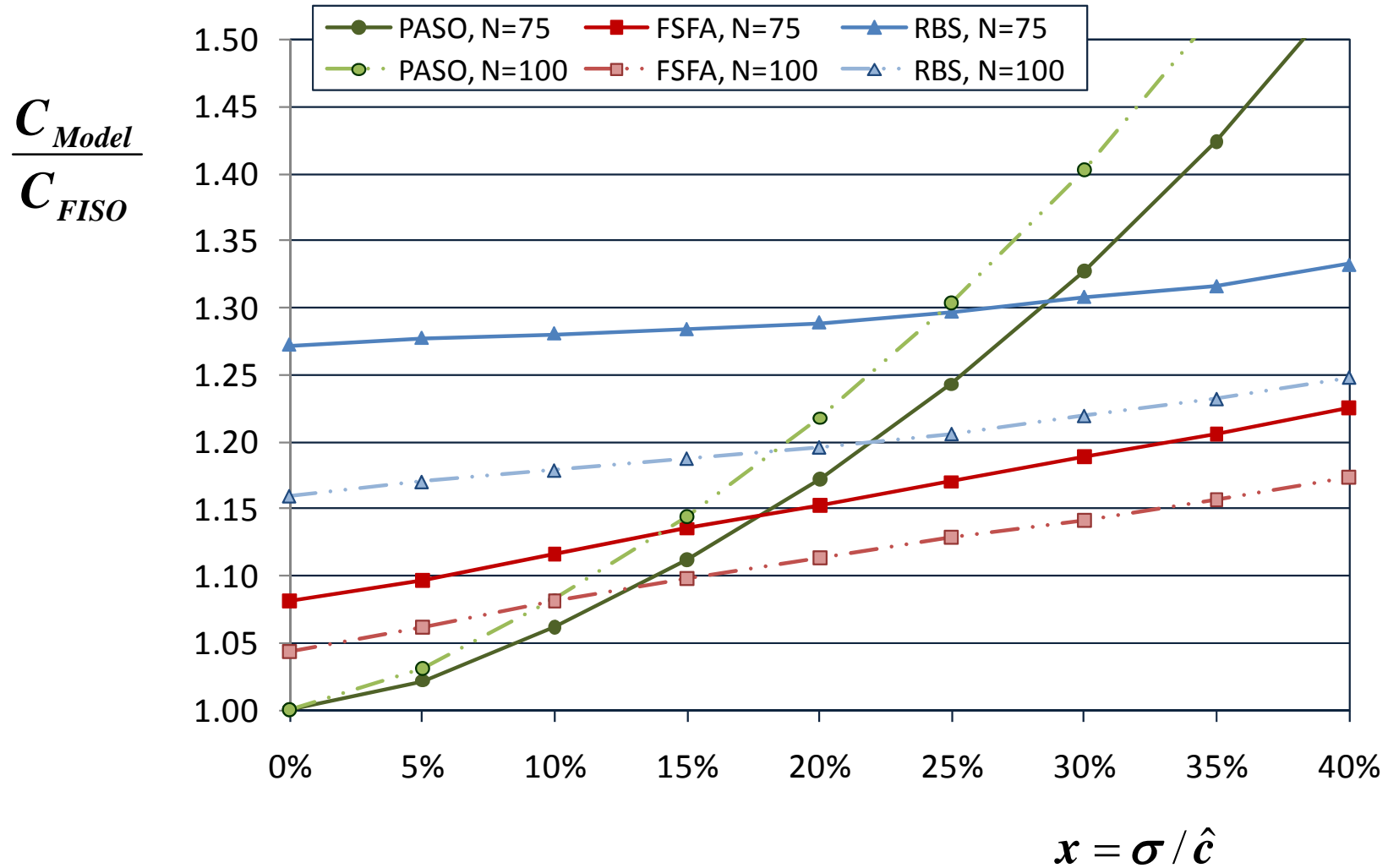
Average α for route r

$$\min_{x_{n,r}, \forall n,r} \hat{C} = \sum_{r=1}^R \left(\rho_r X_r \left(\alpha_{\min} + \theta \sum_{j=2}^r X_{j-1} + \frac{\theta \cdot (X_r + 1)}{2} \right) + g_r \frac{X_r \cdot (X_r + 1)}{2} \right)$$

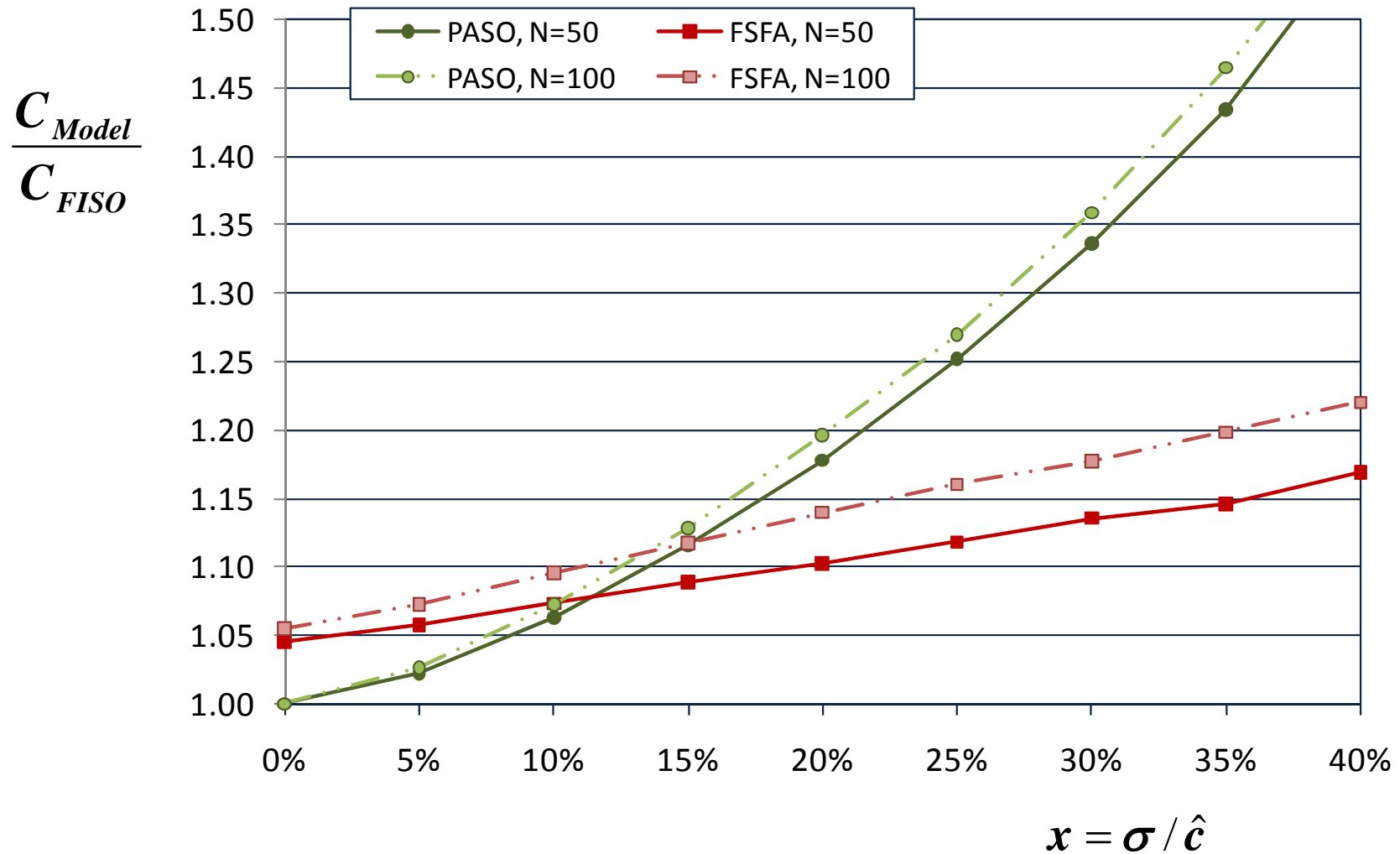
Where: $\theta = \frac{\alpha_{\max} - \alpha_{\min}}{N}$

Constraints: $\sum_{r \in R} X_r = N, X_r \geq 0, \forall r$

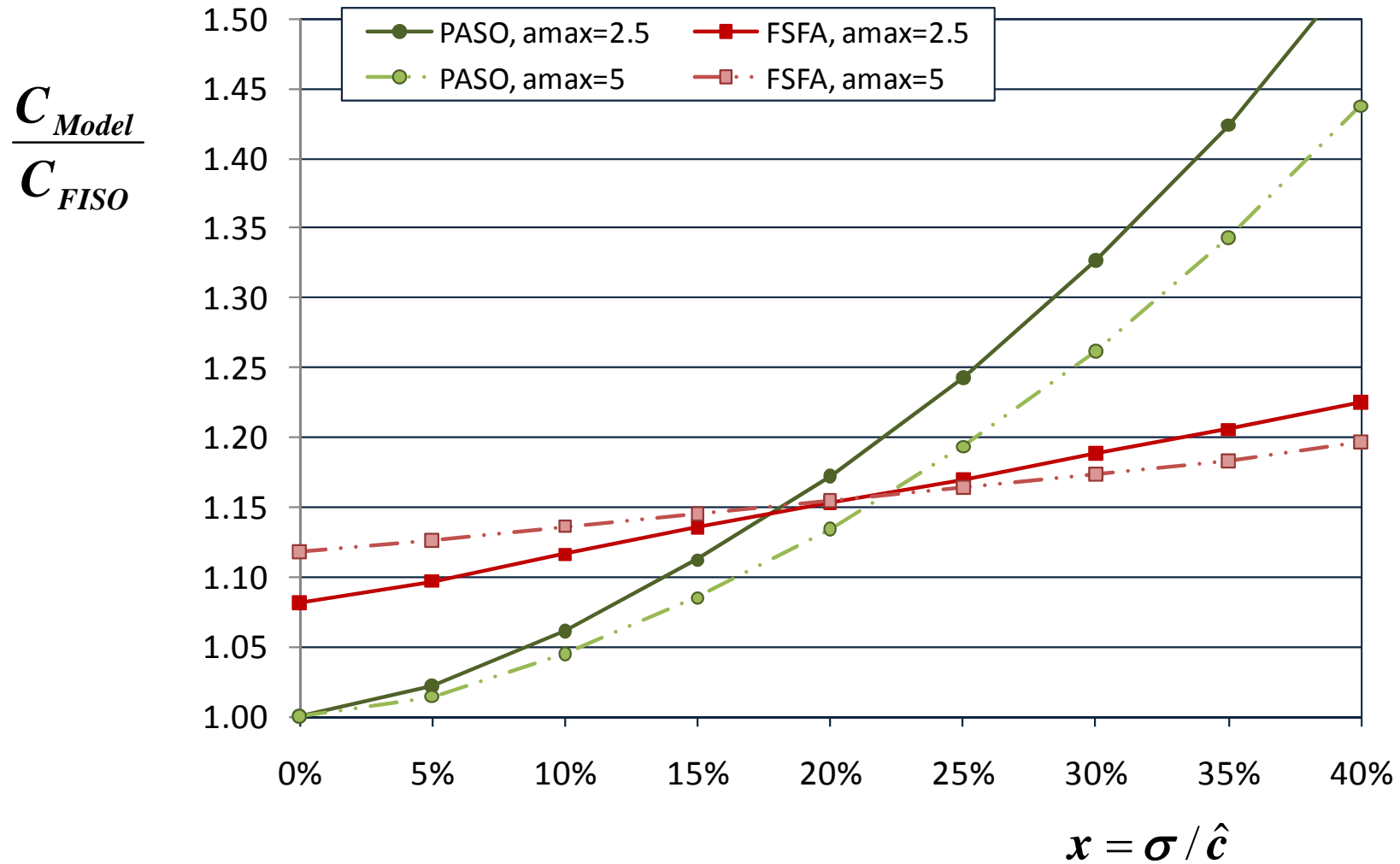
Sensitivity: Changing D_0



Sensitivity: Changing N ($D_0=80$)



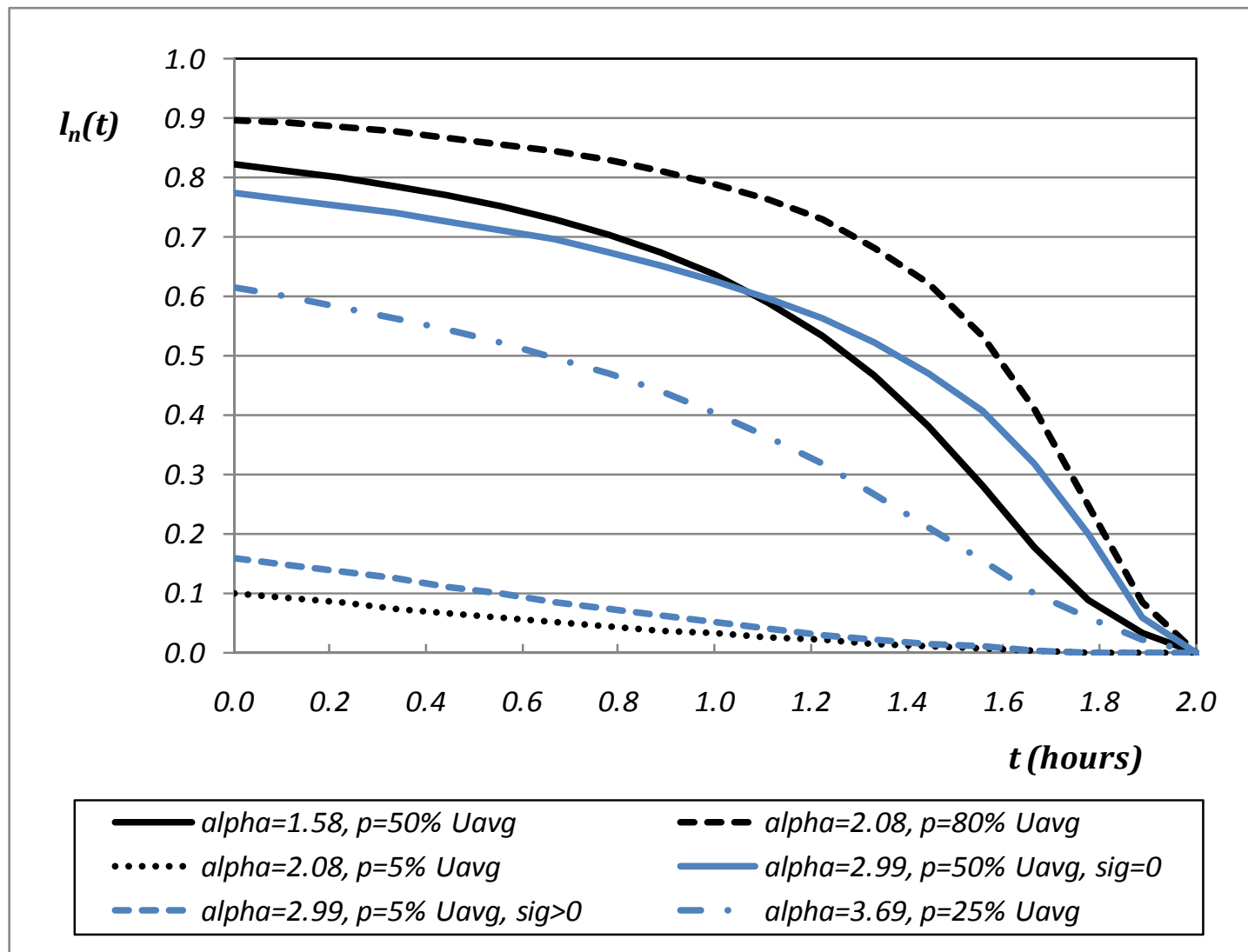
Sensitivity: Changing α



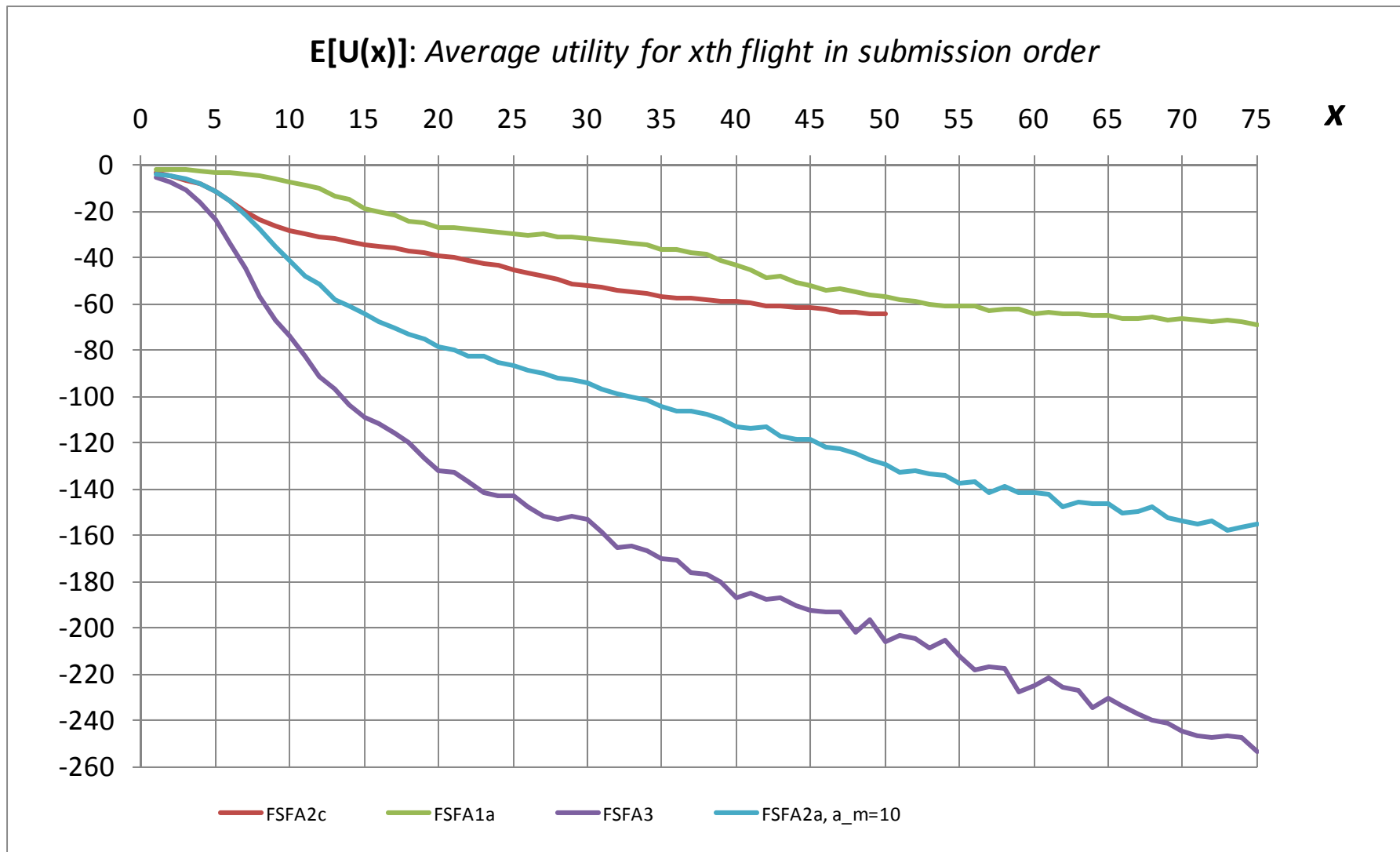
Competition



Utility (as a function of submission time)



Utilities wrt Rank in Submission Order



Model setup & assumptions

- 3 players, 3 resources
- Players desire to win resource of highest utility to them
- Players submit inputs during planning period, $t \in [0, T]$
- Each player knows its own function describing their utility of being x th in submission order (type)
 - Players know their own type but not others
 - Players have general beliefs about types in the population
 - Players' types distributed identically (symmetric game)

Model setup & assumptions (cont'd)

- Expected payoff:

$$E[\pi] = E[U(1st)] * P(1st) + E[U(2nd)] * P(2nd) - \text{sub. time}$$

- Resource utilities and bids linear with respect to submission order and changing conditions, respectively
- Each flight (rather than airline) is a player
- Simultaneous game vs. multi-object descending price auction

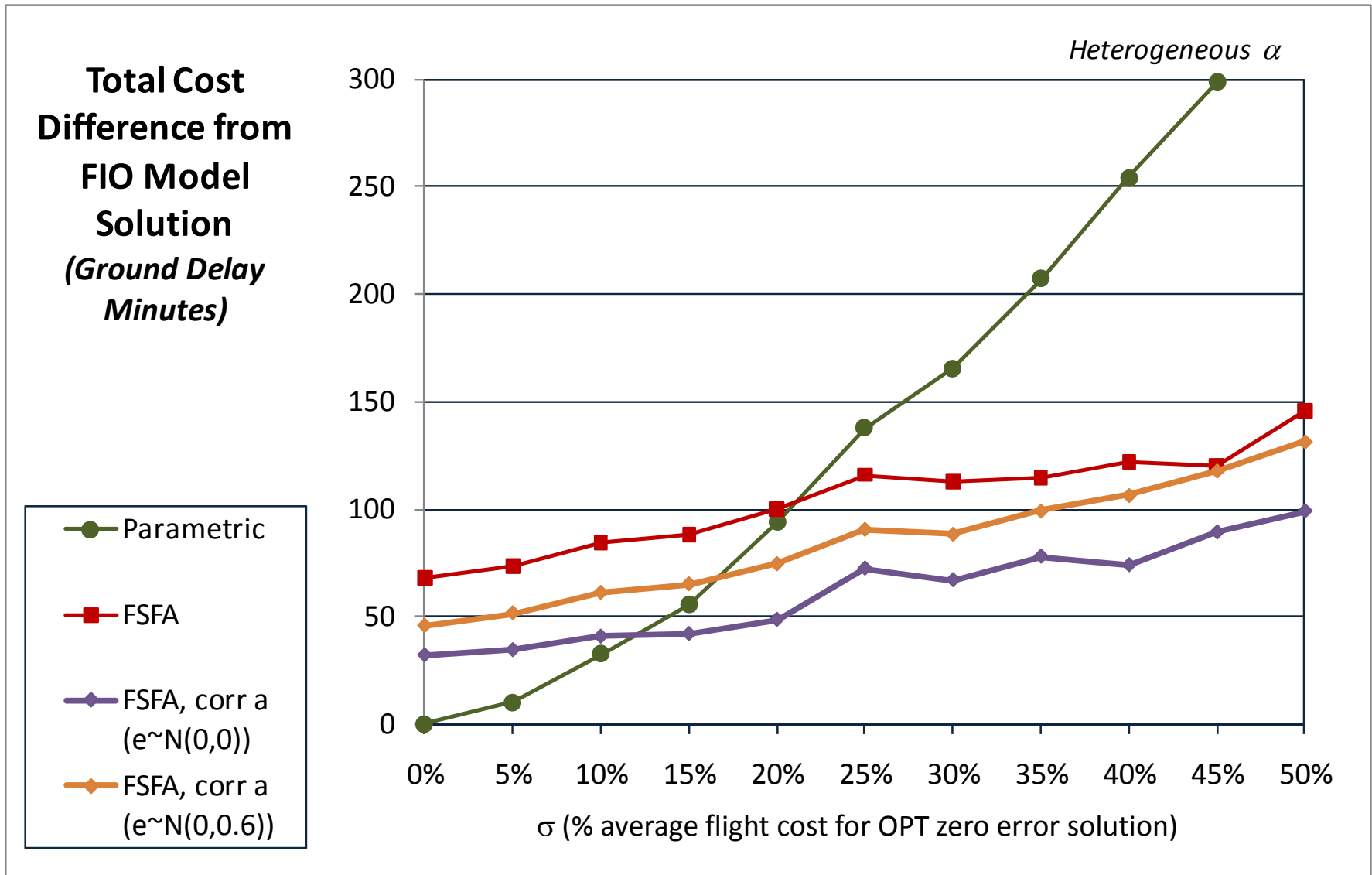
Equity and stuff



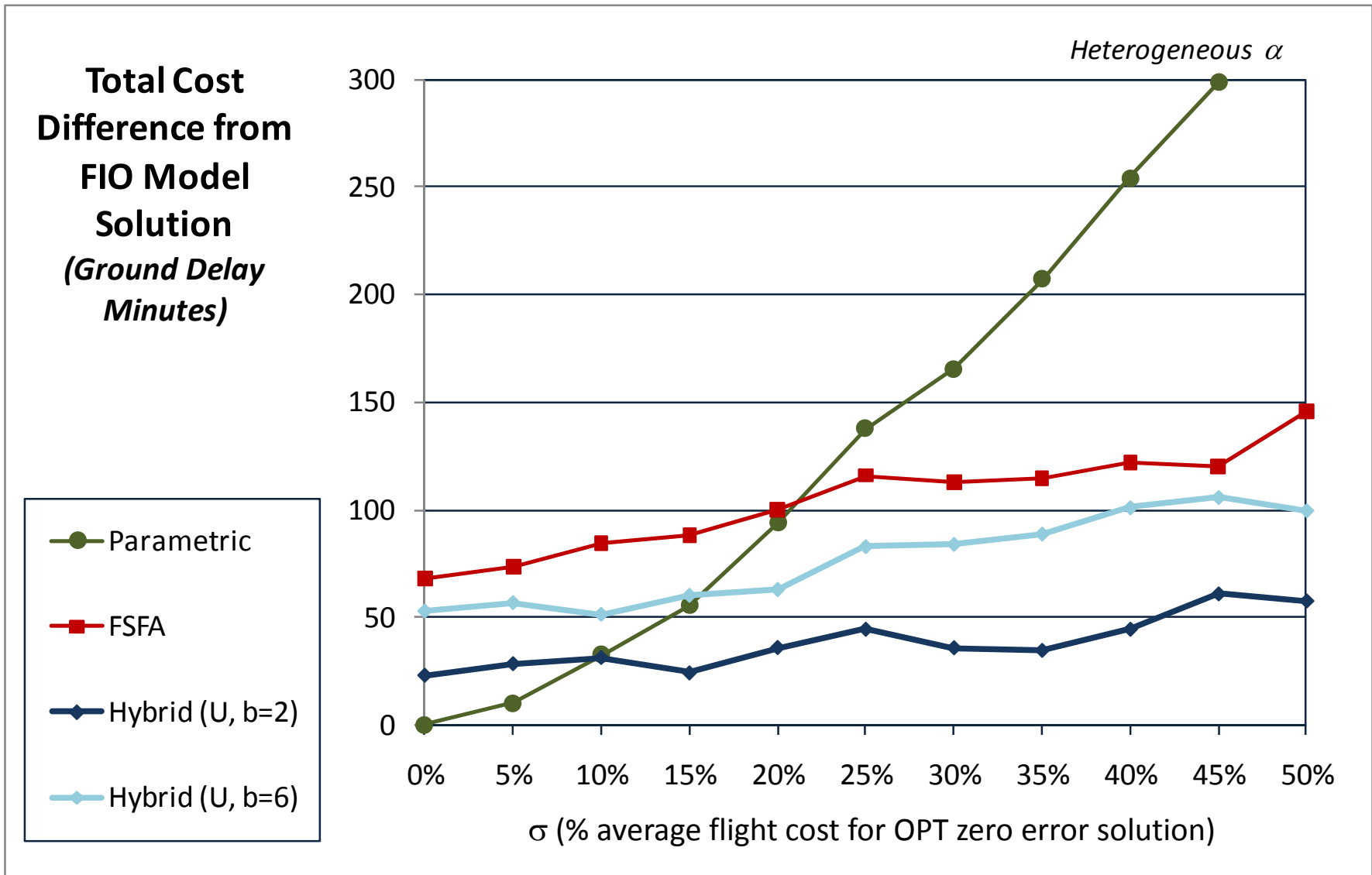
Equity

- FAA has obligation to be fair to airlines (and their customers)
- Necessary to encourage airline participation in CDM
 - Effective ATM highly dependent on accurate operational information that only airlines can provide
 - Best chance of passing along benefits to passengers
- True system efficiency can only be obtained through fair allocation practices (Young, 1994)
 - Balanced/synonymous with efficiency

Case 2: Different scheduled departure times ($s_1 \neq s_2 \neq \dots \neq s_N$)



Case 2: Different scheduled departure times ($s_1 \neq s_2 \neq \dots \neq s_N$)



Ration by Schedule (RBS): introduction

- Well-accepted method of airport arrival slot allocation in Ground Delay Programs (GDPs)
- Flights are allocated best available resources in order of original scheduled departure times at Fix A
- FAA announced AFP, asks users to submit delay thresholds by some time to receive an RBS resource allocation.
- Expect that RBS would incentivize large majority of users to submit.

RBS: properties

- RBS demonstrated to result in an inferior total cost solution compared to FSFA
- May be analogous to bin-packing problem (worst-case best-fit increasing)?
 - Along this rationale, reverse RBS (RRBS) = best-fit decreasing?
 - FSFA = online best-fit?

System Optimal Models (cont'd)

