Converting Technology to Mitigate Environmental Damage

Maurice D. Levi
Faculty of Commerce and Business Administration
University of British Columbia
2053 Main Mall
Vancouver, BC, Canada V6T 1Z2
Tel (604) 822-8260; Fax (604) 822-4695
levi@commerce.ubc.ca.

and

Barrie R. Nault
Haskayne School of Business
University of Calgary
2500 University Drive NW
Calgary, Alberta, Canada
Tel (403) 220-2742; Fax (403) 282-0095
nault@ucalgary.ca

November 9, 2002

We thank Jim Brander, Al Dexter and Alan Kraus for helpful comments. We are also indebted to the Departmental Editor, an Associate Editor, and two referees for their detailed and insightful suggestions. Support was provided by the Natural Sciences and Engineering Research Council of Canada, and the Social Sciences and Humanities Research Council of Canada.
9 Appendix

9.1 Proofs of Lemmas and Theorems

Proof of Lemma 1: Using (1) \( \frac{\partial \Psi(x, s_x, \theta)}{\partial s_x} = 1 \). By writing \( \Psi(x, s_x, \theta) = 0 \) as an optimal output function, \( x = x(s_x, \theta) \), from the implicit function rule

\[
\frac{\partial x(s_x, \theta)}{\partial s_x} = -\frac{\partial \Psi(x, s_x, \theta)}{\partial x}/\partial s_x.
\]

Using (2), we have \( \frac{\partial x(s_x, \theta)}{\partial s_x} > 0 \). QED

Proof of Lemma 2: From (1) and the implicit function rule

\[
\frac{\partial x(s_x, \theta)}{\partial \theta} = -\frac{\partial \Psi(x, s_x, \theta)}{\partial x}/\partial \theta.
\]

From (2), \( \frac{\partial \Psi(x, s_x, \theta)}{\partial x} < 0 \). As for \( \frac{\partial \Psi(x, s_x, \theta)}{\partial \theta} \), we have

\[
\frac{\partial \Psi(x, s_x, \theta)}{\partial \theta} = \frac{\partial^2 PR(x, \theta)}{\partial x \partial \theta} - \frac{\partial^2 C(x, \theta)}{\partial x \partial \theta} < 0
\]

from Assumptions 5 and 7. This means \( \frac{\partial x(s_x, \theta)}{\partial \theta} < 0 \). QED

Proof of Lemma 3: For these firms \( \frac{\partial \Psi(x, \theta)}{\partial \theta} = \frac{\partial^2 PR(x, \theta)}{\partial x \partial \theta} \), which from Assumption 5 is negative. Therefore, from the implicit function rule and (4), \( x'(\theta) < 0 \). QED

Proof of Theorem 1: The perfectly discriminating subsidy, \( S(\theta) \), would equalize a firm’s profit under the conversion and no conversion options:

\[
PR(x(s_x, \theta), \theta) - C(x(s_x, \theta), \theta) + S(\theta) = PR(x(\theta), \theta).
\]

Rearranging and differentiating with respect to \( \theta \), eliminating the first-order conditions (1) and (3), yields

\[
S'(\theta) = \frac{\partial PR(x(\theta), \theta)}{\partial \theta} - \frac{\partial PR(x(s_x, \theta), \theta)}{\partial \theta} + \frac{\partial C(x(s_x, \theta), \theta)}{\partial \theta}.
\]

From the first-order conditions (1) and (3), and from the fact that the marginal subsidy is zero, if conversion costs are increasing in output then \( x(\theta) > x(s_x, \theta)_{s_x=0} \). Thus, from Assumption 8, \( S'(\theta) > 0 \). Otherwise \( x(\theta) \leq x(s_x, \theta)_{s_x=0} \), and from Assumptions 5 and 6, \( S'(\theta) > 0 \). As a result, firms will misrepresent themselves through output. QED
Proof of Theorem 2: Using the envelope theorem with (5) yields
\[
\frac{\partial \phi(S, s_x, \tilde{\theta})}{\partial \theta} = \frac{\partial PR(x(s_x, \tilde{\theta}), \tilde{\theta})}{\partial \theta} - \frac{\partial PR(x(\tilde{\theta}), \tilde{\theta})}{\partial \theta} - \frac{\partial C(x(s_x, \tilde{\theta}), \tilde{\theta})}{\partial \theta}.
\] (17)

The last term is negative from Assumption 6. If the subsidy is large enough to encourage production or if costs of conversion are decreasing in output, then from Assumption 5 the first two terms together are negative, and (17) is negative. Otherwise the combination of the first two terms is positive. Using Assumption 8 the last term dominates the first two terms, and (17) is negative. Thus, firms with plant and equipment condition \( \theta < \tilde{\theta}(S, s_x) \) convert and firms with condition \( \theta > \tilde{\theta}(S, s_x) \) do not. \( \text{QED} \)

Proof of Lemma 4: We can calculate these effects from the condition that defines the firm that is indifferent about converting, \( \phi(S, s_x, \tilde{\theta}) = 0 \). From (17) and the subsequent argument, \( \frac{\partial \phi(S, s_x, \tilde{\theta})}{\partial \theta} < 0 \), while from (5) directly, \( \frac{\partial \phi(s_x, \tilde{\theta})}{\partial s_x} = 1 \). We can write the subsidy schedule in (5) as \( s(x(s_x, \tilde{\theta})) = \int_0^{x(s_x, \tilde{\theta})} s_x dx \), and because the entire schedule is affected, \( \frac{\partial s(x(s_x, \tilde{\theta}))}{\partial s_x} = x(s_x, \tilde{\theta}) + s_x \frac{\partial x(s_x, \tilde{\theta})}{\partial s_x} \). Therefore, using (1) we can cancel terms and write \( \frac{\partial \phi(s_x, \tilde{\theta})}{\partial s_x} = x(s_x, \tilde{\theta}) \).

Hence, from the implicit function rule
\[
\frac{\partial \tilde{\theta}(S, s_x)}{\partial S} = -\frac{1}{\partial \phi(S, s_x, \tilde{\theta})/\partial \theta} > 0 \quad \text{and} \quad \frac{\partial \tilde{\theta}(S, s_x)}{\partial s_x} = -\frac{x(s_x, \tilde{\theta})}{\partial \phi(S, s_x, \tilde{\theta})/\partial \theta} > 0.
\]

\( \text{QED} \)

Proof of Lemma 5: Differentiating producer surplus with respect to the uniform lump-sum subsidy, using (1), gives
\[
\frac{\partial PS(S, s_x)}{\partial S} = [PR(x(s_x, \tilde{\theta}), \tilde{\theta}) - C(x(s_x, \tilde{\theta}), \tilde{\theta})] f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S} - \Pi(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S}.
\]

\( \Pi(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) > PR(x(s_x, \tilde{\theta}), \tilde{\theta}) \) because the former is optimal in the absence of the subsidy. Therefore, \( \partial PS(S, s_x)/\partial S < 0 \). \( \text{QED} \)

Proof of Lemma 6: Differentiating, rearranging and using (6),
\[
\frac{\partial PS(S, s_x)}{\partial s_x} = [PR(x(s_x, \tilde{\theta}), \tilde{\theta}) - C(x(s_x, \tilde{\theta}), \tilde{\theta})] f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial s_x} - \Pi(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial s_x} \\
+ \int_0^{\tilde{\theta}(s_x, \tilde{\theta})} [\frac{\partial PR(x(s_x, \theta), \theta)}{\partial x} - \frac{\partial C(x(s_x, \theta), \theta)}{\partial s_x}] \frac{\partial x(s_x, \theta)}{\partial s_x} f(\theta) d\theta \\
= \frac{\partial PS(S, s_x)}{\partial S} x(s_x, \tilde{\theta}) + \beta.
\]
where $\beta$ is the term under integration. From Lemma 5 the first term is negative. Using (1) and Lemma 1, the second term is also negative. Thus, producer surplus is decreasing in $s_x$.

QED

Proof of Lemma 7: Differentiating and using Lemma 4:

$$\frac{\partial Q(S,s_x)}{\partial S} = q(x(s_x, \tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \left[ \frac{\partial \tilde{\theta}(S,s_x)}{\partial S} \right] > 0$$

and

$$\frac{\partial Q_n(S,s_x)}{\partial S} = -q(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \left[ \frac{\partial \tilde{\theta}(S,s_x)}{\partial S} \right] < 0.$$ 

QED

Proof of Lemma 8: Begin with the environmental damage from converting firms. Differentiating,

$$\frac{\partial Q(S,s_x)}{\partial s_x} = \int_{\tilde{\theta}} \frac{\partial q(x(s_x, \theta), \theta)}{\partial x} \frac{\partial x(s_x, \theta)}{\partial s_x} f(\theta) d\theta + q(x(s_x, \tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S,s_x)}{\partial s_x},$$

and rewriting with (6) yields

$$\frac{\partial Q(S,s_x)}{\partial s_x} = x(s_x, \tilde{\theta}) \frac{\partial Q(S,s_x)}{\partial S} + \int_{\tilde{\theta}} \frac{\partial q(x(s_x, \theta), \theta)}{\partial x} \frac{\partial x(s_x, \theta)}{\partial s_x} f(\theta) d\theta$$

where $\sigma$ is the term under integration. From Lemma 7 the first term is positive. From Assumption 2 and Lemma 1 the second term is also positive.

For the aggregate environmental damage from non-converting firms, from the definition of $Q_n(S,s_x)$ and from Lemma 4, $\partial Q_n(S,s_x)/\partial s_x = -q_n(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) [\partial \tilde{\theta}(S,s_x)/\partial s_x] < 0$. QED

Proof of Theorem 3: The firm with the best plant and equipment condition has $\bar{\theta}$. Begin with $S = s(x) = 0$. Define $dS = S(\bar{\theta})$ as in the proof of Theorem 1. Thus, only the firm with $\bar{\theta}$ makes the conversion. The change in welfare is

$$\left. dB \right|_{ds_x=0} = CS'(X(S,s_x)) \frac{\partial X(S,s_x)}{\partial S} dS + \frac{\partial P S(S,s_x)}{\partial S} dS$$

$$- \frac{\partial \omega(\cdot)}{\partial Q_n} \left[ \frac{\partial Q_n(S,s_x)}{\partial S} \right] dS - \frac{\partial \omega(\cdot)}{\partial Q} \left[ \frac{\partial Q(S,s_x)}{\partial S} \right] dS. \quad (18)$$

From Lemma 5, the change in producer surplus is negative. The change in consumer surplus depends on the relative size of outputs under the conversion and no conversion options, and
because the output-based subsidy is assumed to be zero this in turn depends on the marginal conversion cost in (1). Using Lemma 7, environmental damage is reduced. If the condition in the premise holds, then welfare is improved. QED

**Proof of Theorem 4:** Using a total differential, a necessary condition for $S^*$ to be optimal is

$$
\left. dB \right|_{ds_x^*=0} = CS'(X(S^*, s_x^*)) \frac{\partial X(S^*, s_x^*)}{\partial S^*} ds_x^* + \frac{\partial PS(S^*, s_x^*)}{\partial S^*} dS^* - \frac{\partial \omega(\cdot)}{\partial Q_n} \frac{\partial Q_n(S^*, s_x^*)}{\partial S^*} dS^* - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial Q(S^*, s_x^*)}{\partial S^*} dS^* = 0.
$$

Due to the fact that $s_x^*$ applies to the output from converting firms whereas $S^*$ applies to the number of firms that convert we can set

$$
ds^* = x(s_x^*, \tilde{\theta}) dS^*.
$$

With $x(s_x^*, \tilde{\theta})$ being the output of the marginal firm, $dS^*$ is the lump-sum subsidy per unit of output for this firm. This allows us to restate (19):

$$
\left. dB \right|_{ds_x^* = 0} = CS'(X(S^*, s_x^*)) \frac{\partial X(S^*, s_x^*)}{\partial s_x^*} x(s_x^*, \tilde{\theta}) ds_x^* + \frac{\partial PS(S^*, s_x^*)}{\partial s_x^*} x(s_x^*, \tilde{\theta}) ds_x^* - \frac{\partial \omega(\cdot)}{\partial Q_n} \frac{\partial Q_n(S^*, s_x^*)}{\partial s_x^*} x(s_x^*, \tilde{\theta}) ds_x^* = 0.
$$

It is also necessary that for any interior solution of $s^*(x)$, that is, a solution where $s_x^* \geq 0$ with $s_x^* > 0$ for some $x$, the following condition must hold:

$$
\left. dB \right|_{ds_x^* = 0} = CS'(X(S^*, s_x^*)) \frac{\partial X(S^*, s_x^*)}{\partial s_x^*} ds_x^* + \frac{\partial PS(S^*, s_x^*)}{\partial s_x^*} ds_x^* - \frac{\partial \omega(\cdot)}{\partial Q_n} \frac{\partial Q_n(S^*, s_x^*)}{\partial s_x^*} ds_x^* - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial Q(S^*, s_x^*)}{\partial s_x^*} ds_x^* \geq 0,
$$

allowing for the possibility that the constraint concerning $s(x)$ being non-decreasing in output (Corollary of Theorem 1) is binding. Employing substitutions from (8) and (9), and the proofs of Lemmas 6 and 8,
Using the equality condition (19) and $ds_x^* > 0$, noting that $d\bar{S}^* = ds_x^*$ because the former is the lump-sum subsidy per unit of output for the marginal firm, we can eliminate terms to get
$$CS'(X(S^*, s_x^*))\alpha + \beta - \frac{\partial \omega(\cdot)}{\partial \sigma} \geq 0.$$ Writing out all of the terms results in
$$CS'(X(S^*, s_x^*)) \int_{\bar{\theta}}^{\tilde{\theta}(S^*, s_x^*)} \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} f(\theta) d\theta$$
$$+ \int_{\bar{\theta}}^{\tilde{\theta}(S^*, s_x^*)} \left( \frac{\partial PR(x(s_x^*, \theta), \theta)}{\partial x} - \frac{\partial C(x(s_x^*, \theta), \theta)}{\partial s_x^*} \right) \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} f(\theta) d\theta$$
$$- \frac{\partial \omega(\cdot)}{\partial Q} \int_{\bar{\theta}}^{\tilde{\theta}(S^*, s_x^*)} \frac{\partial q(x(s_x^*, \theta), \theta)}{\partial x} \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} f(\theta) d\theta \geq 0,$$
which, using (1), can be rewritten as
$$\int_{\bar{\theta}}^{\tilde{\theta}(S^*, s_x^*)} [CS'(X(S^*, s_x^*)) - s_x^* - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial q(x(s_x^*, \theta), \theta)}{\partial x}] \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} f(\theta) d\theta \geq 0.$$ And finally,
$$\int_{\bar{\theta}}^{\tilde{\theta}(S^*, s_x^*)} [CS'(X(S^*, s_x^*)) - s_x^* - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial q(x(s_x^*, \theta), \theta)}{\partial x}] \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} f(\theta) d\theta \geq 0.$$ From Lemma 1, this condition requires that $CS'(X(S^*, s_x^*)) - s_x^* - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial q(x(s_x^*, \theta), \theta)}{\partial x} \geq 0$. But, the sufficient condition requires $\frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial q(x(s_x^*, \theta), \theta)}{\partial x} > CS'(X(S^*, s_x^*))$, which implies $s_x^* < 0$, a contradiction.

Now suppose that the optimal lump-sum subsidy is zero, $dB \big|_{ds_x \neq 0} \leq 0$. Then a positive output-based subsidy requires
$$dB \big|_{dS_x = 0} = CS'(X(S^*, s_x^*))\alpha + \beta - \frac{\partial \omega(\cdot)}{\partial Q} \sigma + dB \big|_{ds_x^* = 0} \geq 0$$
which implies
$$CS'(X(S^*, s_x^*))\alpha + \beta - \frac{\partial \omega(\cdot)}{\partial Q} \sigma > -dB \big|_{ds_x^* = 0} > 0.$$ Directly from above, this contradicts the sufficient condition in the Theorem. QED

Proof of Lemma 2e: From (1) and the implicit function rule
$$\frac{\partial x(s_x, \theta, y)}{\partial y} = -\frac{\partial \Psi(x, s_x, \theta, y)/\partial y}{\partial \Psi(x, s_x, \theta, y)/\partial x}.$$
From (2), \( \frac{\partial \Psi(x, s, \theta, y)}{\partial x} < 0 \). As for \( \frac{\partial \Psi(x, s, \theta, y)}{\partial y} \), we have

\[
\frac{\partial \Psi(x, s, \theta, y)}{\partial y} = -\frac{\partial^2 C(x, \theta, y)}{\partial x \partial y} > 0
\]

from Assumption 7e. This means \( \frac{\partial (s, \theta, y)}{\partial y} > 0 \).

**Proof of Lemma 5e:** Redefining producer surplus to include the proportion of firms converting in the output function of firms that convert and the conversion cost, and differentiating with respect to the uniform lump-sum subsidy, using (1), gives

\[
\frac{\partial PS(S, s_x)}{\partial S} = \left[ PR(x(s_x, \tilde{\theta}, y(\cdot), \tilde{\theta}) - C(x(s_x, \tilde{\theta}, y(\cdot), \tilde{\theta}, y(\cdot))) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S} - \Pi(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S} ight.
\]

\[
+ \left. \int_{\tilde{\theta}}^{\tilde{\theta}(S, s_x)} \left[ \left[ \frac{\partial PR(x(\cdot), \theta)}{\partial x} - \frac{\partial C(x(\cdot), \theta, y(\cdot))}{\partial x} \right] \frac{\partial x(\cdot)}{\partial y} - \frac{\partial C(x(\cdot), \theta, y(\cdot))}{\partial y} \right] y'(\cdot) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S} f(\theta) d\theta \right] < 0
\]

where for the purpose of this proof \( x(\cdot) \) is used to represent \( x(s_x, \theta, y(\cdot)) \). As in the proof of Lemma 5, \( \Pi(x(\tilde{\theta}), \tilde{\theta}) f(\tilde{\theta}) > PR(x(s_x, \tilde{\theta}, y(\cdot), \tilde{\theta}) \) because the former is optimal in the absence of the subsidy and therefore the combination of the first two lines is negative. The two terms on the fourth line are positive from (10) and Lemma 4. The third line is the condition in the premise of the lemma where the effects through output are negative from (1) and Lemma 2e, and the effect through conversion costs are positive from Assumption 6e. Hence, \( \frac{\partial PS(S, s_x)}{\partial S} < 0 \). QED

**Proof of Lemma 6e:** Following the same reasoning as Lemma 6, and using the result from Lemma 5e, \( \frac{\partial PS(S, s_x)}{\partial s_x} = [\partial PS(S, s_x)/\partial S] x(s_x, \tilde{\theta}, y(\cdot)) + \beta < 0 \). QED

**Proof of Theorem 1z:** The perfectly discriminating subsidy would equalize profits from the time of conversion, \( \hat{z} \):

\[
\int_{\hat{z}}^{\infty} [PR(x(s_x, \theta), \theta) - C(x(s_x, \theta), \theta, \hat{z})] dz + S(\theta) = \int_{\hat{z}}^{\infty} PR(x(\theta), \theta) dz.
\]

Using the same steps as in the proof of Theorem 1 we get

\[
\int_{\hat{z}}^{\infty} \left[ \frac{\partial PR(x(s_x, \theta), \theta)}{\partial \theta} - \frac{\partial C(x(s_x, \theta), \theta, \hat{z})}{\partial \theta} - \frac{\partial PR(x(\theta), \theta)}{\partial \theta} \right] dz = S'(\theta) > 0,
\]
yielding the same result at Theorem 1 and its corollary. QED

Proof of Theorem 2z: Using the same steps as the proof of Theorem 2, but with (14) instead of (5), we have \( \partial \gamma(s_x, \tilde{\theta}, 0)/\partial \tilde{\theta} < 0 \). Therefore, if \( \tilde{\theta} < \tilde{\theta}_z \), then firms with condition \( \theta \leq \tilde{\theta} \) convert immediately and firms with condition \( \theta > \tilde{\theta} \) do not. QED

Proof of Lemma 9: For firms that convert in \((0, \tilde{z})\) the first-order condition maximizing (13) by choice of \( \hat{z} \) is

\[
PR(x(\theta), \theta) - PR(x(s_x, \theta), \theta) - s(x(s_x, \theta)) + C(x(s_x, \theta), \theta, \hat{z})
- \int_{\hat{z}}^{\infty} \frac{\partial C(x(s_x, \theta), \theta, \hat{z})}{\partial \hat{z}} d\hat{z} = 0 = \xi(\hat{z}, \theta, s_x),
\]

where the term under integration is negative from Assumption 6z. The second-order condition is \( 2 \partial C(x(s_x, \theta), \theta, \hat{z})/\partial \hat{z} - \int_{\hat{z}}^{\infty} \left[ \partial^2 C(x(s_x, \theta), \theta, \hat{z})/\partial \hat{z}^2 \right] d\hat{z} < 0 \), which is true as long as first-order effects dominate second-order effects or if conversion costs are convex in time of conversion. (21) defines \( \hat{z}(s_x, \theta) \). Using the envelope theorem and (17) from the proof of Theorem 2

\[
\frac{\partial \xi(\hat{z}, \theta, s_x)}{\partial \theta} = \frac{\partial PR(x(\theta), \theta)}{\partial \theta} - \frac{\partial PR(x(s_x, \theta), \theta)}{\partial \theta} + \frac{\partial C(x(s_x, \theta), \theta, \hat{z})}{\partial \theta} > 0,
\]

noting that from Assumption 7z the derivative of the term under integration in (21) is zero. From the second-order conditions of (21) and the implicit function rule \( \partial \hat{z}(s_x, \theta)/\partial \theta > 0 \).

Writing the subsidy schedule as in the proof of Lemma 4, and using the envelope theorem, \( \partial \xi(\hat{z}, \theta, s_x)/\partial s_x = x(s_x, \theta) > 0 \). From the implicit function rule, \( \partial \hat{z}(s_x, \theta)/\partial s_x < 0 \). Q.E.D.

Proofs of Lemmas 5-8 Under Strategic Timing: \( \hat{z}(s_x, \hat{\theta}(S, s_x)) = 0 \) and \( \hat{z}(s_x, \hat{\theta}_z(S, s_x)) = \bar{z} \), which affect integration limits. The effect of the uniform lump-sum subsidy on producer surplus is

\[
\frac{\partial PS_z(S, s_x)}{\partial S} = \int_{\tilde{z}}^{\infty} [PR(x(s_x, \tilde{\theta}_z), \tilde{\theta}_z) - C(x(s_x, \tilde{\theta}_z), \tilde{\theta}_z, \tilde{z}) - \Pi(x(\tilde{\theta}_z), \tilde{\theta}_z)] dz f(\tilde{\theta}_z) \frac{\partial \tilde{\theta}_z(S, s_x)}{\partial S} < 0
\]

as in Lemma 5. The effect of the output-based subsidy on producer surplus is

\[
\frac{\partial PS_z(S, s_x)}{\partial s_x} = \frac{\partial PS_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) \]

38
where the last line is simplified from (21). The first term on the right hand side is negative from Lemma 5. The effects through output in the second and third terms, which we denote by $\beta_z$, are negative, similar to the proof of Lemma 6. The fourth term representing the effect of earlier conversion, which we denote by $\mu$, is negative from Lemma 9. Therefore,

$$\frac{\partial PS_z(S, s_x)}{\partial s_x} = \frac{\partial PS_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \beta_z + \mu < 0.$$  

The impact of the uniform lump-sum subsidy on aggregate damage from non-converting firms is

$$\frac{\partial Q_{nz}(S, s_x)}{\partial S} = -\int_{\tilde{z}(s_x, \theta)}^\infty q_n(x(\theta), \theta) d\tilde{z}_z x(s_x, \tilde{\theta}_z) d\tilde{z}_z(S, s_x) < 0,$$

and on aggregate damage from converting firms is

$$\frac{\partial Q_z(S, s_x)}{\partial S} = \int_{\tilde{z}(s_x, \theta)}^\infty q(x(s_x, \theta), \theta) d\tilde{z}_z x(s_x, \tilde{\theta}_z) d\tilde{z}_z(S, s_x) > 0.$$  

The impact of the output-based subsidy on aggregate damage from non-converting firms is

$$\frac{\partial Q_{nz}(S, s_x)}{\partial s_x} = \frac{\partial Q_{nz}(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \int_{\theta(S, s_x)}^\infty q_n(x(\theta), \theta) \frac{\partial \tilde{z}_z(S, \theta)}{\partial s_x} f(\theta) d\theta$$

$$= \frac{\partial Q_{nz}(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \nu_n < 0,$$

and on aggregate damage from converting firms is

$$\frac{\partial Q_z(S, s_x)}{\partial s_x} = \frac{\partial Q_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \int_{\theta(S, s_x)}^\infty q(x(s_x, \theta), \theta) \frac{\partial x(s_x, \theta)}{\partial s_x} d\tilde{z}_z f(\theta) d\theta$$

$$+ \int_{\theta(S, s_x)}^\infty q(x(s_x, \theta), \theta) \frac{\partial x(s_x, \theta)}{\partial s_x} d\tilde{z}_z f(\theta) d\theta - \int_{\theta(S, s_x)}^\infty q(x(s_x, \theta), \theta) \frac{\partial \tilde{z}_z(S, \theta)}{\partial s_x} f(\theta) d\theta.$$  

The first term on the right hand side is positive from Lemma 7. The effects through output in the second and third terms, which we denote by $\sigma_z$, are positive, similar to the proof of
Lemma 8. The last term representing the effect of earlier conversion, which we denote by \( \nu \), is positive from Lemma 9. Therefore,

\[
\frac{\partial Q_z(S, s_x)}{\partial s_x} = \frac{\partial Q_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \sigma_z + \nu > 0.
\]

\( QED \)

Proof of Theorem 4z: Following the proof of Theorem 4, the marginal firm is still \( \tilde{\theta}_z \) but (20) now includes expanded versions of \( \partial X_z(S, s_x)/\partial s_x \), \( \partial PS(S, s_x)/\partial s_x \), \( \partial Q_{nz}(S, s_x)/\partial s_x \), and \( \partial Q_z(S, s_x)/\partial s_x \). After removing the effects of (19) we can eliminate terms to get

\[
CS'(X_z(S^*, s_x^*)) \alpha_z + CS'(X_z(S^*, s_x^*)) \eta + \beta_z + \mu - \frac{\partial \omega(\cdot)}{\partial Q_{nz}} \nu_n - \frac{\partial \omega(\cdot)}{\partial Q_z} \sigma_z - \frac{\partial \omega(\cdot)}{\partial Q_z} \nu 
\geq 0
\]

from (16) and the proofs of Lemmas 5-8 under strategic timing. Restating after using the proof of Theorem 4 and substitutions

\[
\int_0^{\tilde{\theta}(S^*, s_x^*)} \int_0^\infty [CS'(X_z(S^*, s_x^*)) - s_x^* - \frac{\partial \omega(\cdot)}{\partial Q_z} s_x^*] \frac{\partial x(s_x^*, \theta)}{\partial s_x^*} dz f(\theta) d\theta
\]

The first two lines are nonnegative from the first part of the premise of Theorem 4z. The last line is nonnegative from the second part of the premise of Theorem 4z. \( QED \)

9.2 The Tax Case

In the case of taxes, firms that convert pay the cost of conversion and those that do not are taxed. The profit function for the converting firms is

\[
PR(x, \theta) - C(x, \theta).
\]

The first-order condition for maximizing profit by choosing level of output is

\[
\frac{\partial PR(x, \theta)}{\partial x} - \frac{\partial C(x, \theta)}{\partial x} = 0.
\]
The second-order conditions follow from Assumptions 4 and 6. The first-order condition implicitly defines output as a function of the firm’s plant and equipment condition and \(x'(\theta) > 0\). The profit function for firms that do not convert is

\[ PR(x, \theta) - T - t(x) \]

where \(T\) is the uniform lump-sum tax and \(t(x)\) is the nonlinear tax as a function of output with the boundary \(t(0) = 0\). The first-order condition is

\[ \frac{\partial PR(x, \theta)}{\partial x} - t_x = 0, \]

where \(t_x\) is the marginal tax and the second-order condition follows from Assumption 4 with reasonable conditions on \(t_x'\). The first-order condition defines \(x(t_x, \theta)\), which is decreasing in both arguments.

Using similar reasoning as in their proofs in the text involving misrepresentation, Theorem 1 and its Corollary continue to hold in the tax case: the first-best incentive program is infeasible and is constrained to \(t_x \geq 0\). We define \(\tilde{\theta}(T, t_x)\) by the equation

\[ PR(x(\tilde{\theta}), \tilde{\theta}) - C(x(\tilde{\theta}), \tilde{\theta}) - PR(x(t_x, \tilde{\theta}), \tilde{\theta}) + T + t(x(t_x, \tilde{\theta})) = 0 = \phi(T, t_x, \tilde{\theta}). \]

Using reasoning similar to that employed in the proof, the separation result of Theorem 2 holds in this case also: \(\theta \in [\underline{\theta}, \bar{\theta}]\) convert and \(\theta \in (\tilde{\theta}, \bar{\theta}]\) do not. Use of the implicit function rule gives the relation

\[ \frac{\partial \tilde{\theta}(T, t_x)}{\partial t_x} = x(t_x, \tilde{\theta}) \frac{\partial \tilde{\theta}(T, t_x)}{\partial T}. \]

Dropping the arguments of \(\tilde{\theta}\), aggregate output, producer surplus, and environmental damage from non-converting and converting firms are

\[ X(T, t_x) = \int_{\underline{\theta}}^{\tilde{\theta}(T,t_x)} x(\theta)f(\theta)d\theta + \int_{\tilde{\theta}(T,t_x)}^{\bar{\theta}} x(t_x, \theta)f(\theta)d\theta, \]

\[ PS(T, t_x) = \int_{\underline{\theta}}^{\tilde{\theta}(T,t_x)} [PR(x(\theta), \theta)) - C(x(\theta), \theta)]f(\theta)d\theta + \int_{\tilde{\theta}(T,t_x)}^{\bar{\theta}} PR(x(t_x, \theta), \theta)f(\theta)d\theta, \]

\[ Q_n(T, t_x) = \int_{\bar{\theta}(T,t_x)}^{\bar{\theta}} q_n(x(t_x, \theta), \theta)f(\theta)d\theta \]

and

\[ Q(T, t_x) = \int_{\underline{\theta}}^{\tilde{\theta}(T,t_x)} q(x(\theta), \theta)f(\theta)d\theta, \]
respectively. Differentiation and substitution yields
\[
\frac{\partial X(T, t_x)}{\partial t_x} = \frac{\partial X(T, t_x)}{\partial T} x(t_x, \tilde{\theta}) + \int_\theta^\tilde{\theta} \frac{\partial x(t_x, \theta)}{\partial t_x} f(\theta) d\theta,
\]
\[
\frac{\partial PS(T, t_x)}{\partial t_x} = \frac{\partial PS(T, t_x)}{\partial T} x(t_x, \tilde{\theta}) + \int_\theta^\tilde{\theta} \frac{\partial PR(x(t_x, \theta), \theta)}{\partial x} \frac{\partial x(t_x, \theta)}{\partial t_x} f(\theta) d\theta,
\]
\[
\frac{\partial Q_n(T, t_x)}{\partial t_x} = \frac{\partial Q_n(T, t_x)}{\partial T} x(t_x, \tilde{\theta}) + \int_\theta^\tilde{\theta} \frac{\partial q_n(x(t_x, \theta), \theta)}{\partial x} \frac{\partial x(t_x, \theta)}{\partial t_x} f(\theta) d\theta
\]
and
\[
\frac{\partial Q(T, t_x)}{\partial t_x} = \frac{\partial Q(T, t_x)}{\partial T} x(t_x, \tilde{\theta}).
\]

Social welfare is defined as
\[
B(T, t_x) = CS(X(T, t_x)) + PS(T, t_x) - \omega(Q_n(T, t_x), Q(T, t_x)).
\]

Theorem 3 from the text also holds here, noting from the proof that because output is not necessarily reduced in the tax case, consumer surplus may be increased by increases in \(T\).

We now deliver the main result for the tax case.

**Theorem 4 (tax case):** A sufficient condition for the optimal tax to be a uniform lump-sum is that the marginal environmental damage of non-converting firms must be no more than the marginal consumer surplus.

**Proof:** Let \(T^*\) and \(t_x^*\) be the optimal settings of the uniform lump-sum and variable schedule of taxes from the control program that maximizes \(B(T, t_x)\). The proof follows the same structure as that of Theorem 4 in the text, finally yielding the equation
\[
\int_\theta^\tilde{\theta} [CS'(X(T^*, t_x^*)) + t_x^* - \frac{\partial \omega(\cdot)}{\partial Q_n} \frac{\partial q_n(x(t_x^*, \theta), \theta)}{\partial x} \frac{\partial x(t_x^*, \theta)}{\partial t_x^*}] f(\theta) d\theta \geq 0.
\]
This implies \(CS'(X(T^*, t_x^*)) + t_x^* - \frac{\partial \omega(\cdot)}{\partial Q_n} \frac{\partial q_n(x(t_x^*, \theta), \theta)}{\partial x} \leq 0\), which together with the sufficient condition in the theorem requires \(t_x^* < 0\), a contradiction. QED