1. Proofs

PROOF OF PROPOSITION 1

For the M/G/1/K/PS queue, the probability of having $n$ sessions in the site is (Sevcik and Mitrani 1981),

$$p_n = \frac{1 - \rho}{1 - \rho^{K+1}} \rho^n,$$

for $n = 0, 1, \Lambda , K$, where $\rho = \lambda / \mu$. This result requires the mean service rate only, independent of the service time distribution. The balking probability can be found when $n = K$, explicitly,

$$p_K = \frac{1 - \rho}{1 - \rho^{K+1}} \rho^K.$$

Therefore, the proportion of arrivals that get serviced is $\lambda_s = (1 - p_K) \lambda$. The average number of sessions handled, $N_K$, can be obtained as,

$$N_K = \sum_{n=0}^{K} np_n = \frac{\rho}{1 - \rho} - \frac{(K+1)\rho^{K+1}}{1 - \rho^{K+1}}.$$

The loss due to customer impatience can be estimated as follows. We tag sessions in the system whose owners (the corresponding customers who submit those sessions) have left the site. Note that the leaving of customers is not observable by the server, so the required processing will still be done for sessions whose owners have quit. Suppose that out of $N_K$ sessions, a proportion $l$ are tagged. So a total of $lN_K$ are tagged, while $(1 - l)N_K$ are not. Now consider an elapsed of time $\Delta t$, $\lambda_s \Delta t$ new sessions (or users) will come to the system and $\lambda_s \Delta t$ leave the system (a processing rate of $\mu$, with a probability of $\lambda_s / \mu$ being busy). The change in the number of tagged sessions is,

$$l(N_K - \lambda_s \Delta t) + ((1-l)(N_K - \lambda_s \Delta t) + \lambda_s \Delta t) \nu \cdot \Delta t.$$

The first term is the number of initially tagged sessions remaining the system, while the second term is the number of untagged sessions that become tagged during $\Delta t$. In the steady state, the number of tagged sessions should be $lN_K$. Taking the limit $\Delta t \rightarrow 0$, we find that,
\[ l = \frac{vN_K}{\lambda_s + vN_K}. \]

**PROOF OF PROPOSITION 2**

Substituting the expression for the advertising cost, we construct the Hessian matrix for the expected profit,

\[
\pi_{\lambda\lambda} = \frac{\partial^2 \pi}{\partial \lambda^2} = \pi_{\mu\mu} - a(2\lambda / \lambda_s - 1) = -\frac{2hv(\mu + \nu)}{(\mu - \lambda + \nu)^3} - a(2\lambda / \lambda_s - 1),
\]

\[
\pi_{\mu\mu} = \frac{\partial^2 \pi}{\partial \mu^2} = \frac{2hv\lambda}{(\mu - \lambda + \nu)^3};
\]

\[
\pi_{\lambda\mu} = \pi_{\mu\lambda} = \frac{\partial^2 \pi}{\partial \lambda \partial \mu} = \frac{hv(\mu + \lambda + \nu)}{(\mu - \lambda + \nu)^3}.
\]

It is obvious that \( \pi_{\mu\mu} < 0 \). We next examine the determinant of the Hessian matrix,

\[
det H = \pi_{\lambda\lambda}\pi_{\mu\mu} - \pi_{\lambda\mu}\pi_{\mu\lambda} = -\pi_{\mu\mu} \left( -\frac{hv}{2(\mu - \lambda + \nu)^2} + \frac{a(2\lambda / \lambda_s - 1)}{(\lambda(1 - \lambda / \lambda_s))^2} \right). \tag{A1}
\]

The condition described by Proposition 2 results from the Hessian being negative semi-definite, that is, \( \det H \geq 0 \). In the region where \( \det H < 0 \), the profit function displays a saddle shape, and a direction of increase in the profit function will always exist until the boundary is reached.

**PROOF OF PROPOSITION 3**

Assume that the interior maximum exists. The first order condition gives,

\[
\frac{\partial \pi}{\partial \lambda} = h\left(1 - \frac{v(\mu + \nu)}{(\mu - \lambda + \nu)^2}\right) - \frac{a\lambda_s}{\lambda(\lambda_s - \lambda)} = 0; \tag{A2}
\]

\[
\frac{\partial \pi}{\partial \mu} = h\frac{v\lambda}{(\mu - \lambda + \nu)^2} - \gamma_1 = 0. \tag{A3}
\]

Equation A3 renders Equation 4 in Proposition 3. The optimal \( \lambda \) can be solved by,

\[
R(\lambda) = \frac{a\lambda_s}{\lambda(\lambda_s - \lambda)} - h - \gamma_1 - \frac{hv\gamma_1}{\lambda} = 0,
\]

in the region \( \lambda_s / 2 < \lambda < \lambda_s \) where the profit function is concave. It can be shown that the function \( R(\lambda) \) is strictly convex in \( \lambda \). Its first derivative,

\[
\frac{dR(\lambda)}{d\lambda} = -\frac{1}{2} \frac{h v \gamma_1}{\lambda} \frac{1}{\lambda} + \frac{a(2\lambda / \lambda_s - 1)}{(\lambda(1 - \lambda / \lambda_s))^2},
\]

is negative at \( \lambda = \lambda_s / 2 \), and increases to infinity at \( \lambda = \lambda_s \). Suppose that \( dR(\lambda) / d\lambda \) crosses zero at \( \lambda = \lambda_1 \) which gives the minimum of \( R(\lambda) \). Therefore, if \( R(\lambda_1) < 0 \), there are at most two roots for \( R(\lambda) \) in the region \( \lambda_s / 2 < \lambda < \lambda_s \), one above \( \lambda_1 \) and one below. If \( R(\lambda_1) = 0 \), there is only one (degenerate) root at \( \lambda = \lambda_1 \). If \( R(\lambda_1) > 0 \), the solution does not exist.

Notice that, given Equation 4 or A3, \( dR(\lambda) / d\lambda \) is nothing but \( \det H / (\pi_{\mu\mu}) \). If there are two roots, the larger gives a positive \( dR(\lambda) / d\lambda \), therefore is a maximum. Equation 5 in
Proposition 3 follows. The situation where \( R(\lambda_1) = 0 \) corresponds to an optimal solution residing on the boundary defined in Proposition 2 by \( \det H = 0 \) that is equivalent to \( dR(\lambda_1)/d\lambda = 0 \).

**Proof of Corollary 2**

The case where \( F = \pi^* \) results trivially from the envelop theorem. Taking the derivative of Equations A2 and A3 with respect to \( a \), we have,

\[
\frac{\partial \lambda^*}{\partial a} = \frac{\lambda}{\lambda - \lambda^*} \left( \frac{\gamma_1 - \gamma_0}{\lambda^*} \right) < 0;
\]
\[
\frac{\partial \mu^*}{\partial a} = -\frac{\lambda}{\lambda - \lambda^*} \left( \frac{\gamma_1 - \gamma_0}{\lambda^*} \right) < 0.
\]

Similarly, \( \partial \lambda^* / \partial \gamma_1 = -\lambda / \det H - 0 \) and \( \partial \mu^* / \partial \gamma_1 = \lambda / \det H < 0 \).

**Proof of Proposition 4**

The e-tailer’s optimization problem can be rewritten with a Lagrangian,

\[
L = S(\lambda, \mu) - (\gamma_0 + \gamma_1 \mu + A) + \xi \left( \frac{\lambda}{1 + (\lambda / \lambda^* - 1) \exp(-A/a)} - \lambda \right),
\]

where \( \xi \geq 0 \) is the Lagrangian multiplier. The first order condition gives,

\[
\frac{\partial L}{\partial \lambda} = h \left( 1 - \frac{v(\mu + v)}{(\mu - \lambda + v)^2} \right) - \xi;
\]
\[
\frac{\partial L}{\partial \mu} = h \left( \frac{\lambda}{(\mu - \lambda + v)^2} \right) - \gamma_1;
\]

which, when set to zero, give \( \lambda_M \) and \( \mu^* \) that are explicitly expressed in Proposition 4. It can be easily observed that the condition \( h - \xi - \gamma_1 < 0 \) must hold such that \( \lambda < \mu \).

The derivative with respect to \( A \) is,

\[
\frac{\partial L}{\partial A} = -1 + \frac{\xi \lambda}{A(1 + (\lambda / \lambda^* - 1) \exp(-A/a))^2}.
\]

It has two roots for \( A \). We can convert these roots to corresponding demand levels, explicitly,

\[
\lambda = \frac{\lambda}{2} \left( 1 \pm \sqrt{1 - \frac{4a}{\xi \lambda}} \right),
\]

where \( \xi \geq 4a / \lambda \). If \( \xi < 4a / \lambda \), \( \partial L / \partial A < 0 \), therefore E-tailers will not advertise \( (A = 0) \).

Proposition 2 identifies the expression with positive sign, which we denote as \( \lambda_M \), as the maximum point. The Lagrange multiplier, \( \xi \), can be obtained by setting \( \lambda_M = \lambda_M \).

**Lemma 1.** Define a function, \( g(\xi) = \lambda_M - \lambda_M \). Its first derivative is positive at \( \xi = \xi^* \), i.e.,

\[
\frac{\partial g(\xi)}{\partial \xi} \bigg|_{\xi = \xi^*} > 0,
\]

where \( \xi^* \) is the solution to \( g(\xi) = 0 \).

**Proof.** The function \( g(\xi) \) is convex, as \( \partial^2 g(\xi) / \partial \xi^2 > 0 \). Therefore, its first derivative, \( \partial g(\xi) / \partial \xi \), is a strictly increasing function of \( \xi \). It varies from \( -\infty \) to \( \infty \) as \( \xi \) increases from \( 4a / \lambda \) to \( h - \gamma_1 \). We denote \( \xi_1 \) the value of \( \xi \) where \( \partial g(\xi_1) / \partial \xi = 0 \). For \( \xi > \xi_1 \), \( g(\xi) \) is strictly
increasing and goes to $\infty$ when $\xi = h - \gamma_i$. If $g(\xi_i) < 0$, $g(\xi)$ will cross zero at $\xi = \xi^* > \xi_i$. Therefore $\partial g(\xi^*) / \partial \xi > 0$, and $g(\xi_i) < 0$ is the condition that a solution exists.

The above inequality is actually identical to Equation A1 if we substitute $\lambda_{IT}$ and $\mu^*$ in the first term of Equation A1 and $\lambda_M$ in the second term. Unsurprisingly, Lemma 1 re-states the concavity condition in terms of $\xi$. In the case where there are two real roots of $g(\xi)$, the smaller one leads to a negative value of $\partial g(\xi) / \partial \xi$, and therefore a negative determinant of the Hessian matrix.

Proof of Corollary 3

It follows Lemma 1.

Proof of Proposition 5

The optimal demand for the marketing can be written as,

$$\lambda_M = \frac{\lambda_{\infty}}{2} \left( 1 + \sqrt{1 - \frac{4a}{(h - d\eta / d\lambda)\lambda_{\infty}}} \right).$$

To equate this expression to the centralized solution $\lambda^*$ given in Proposition 4, we require the first condition in Proposition 4 to hold. To ensure the concavity, we have,

$$-\frac{d^2\eta}{d\lambda^2} \bigg|_{\lambda=\lambda^*} < \frac{a(2\lambda^*/\lambda_{\infty} - 1)}{\lambda^2(\lambda^*/\lambda_{\infty} - 1)^2} = \frac{\xi^2}{\lambda^*} \frac{4a}{\lambda_{\infty}} \left(1 - \frac{4a}{\xi^2\lambda_{\infty}}\right).$$

Further, it is straightforward to show that IT will choose $\lambda_{IT} = \lambda^*$ and $\mu^*$, given the first condition of Proposition 5 holds. Let us check the second order conditions by constructing the Hessian matrix:

$$\pi_{\lambda\lambda} = \frac{\partial^2 \pi}{\partial \lambda^2} = \pi_{0,\lambda\lambda} + \frac{d^2\eta}{d\lambda^2}.$$ 

The rest of matrix elements and $\pi_{0,\lambda\lambda}$ are given in the proof of Proposition 2. The determinant of the Hessian matrix,

$$\det H = -\frac{h^2\nu^2}{(\mu - \lambda + \nu)^4} + \frac{d^2\eta}{d\lambda^2} \pi_{\mu\mu} > 0,$$

so the Hessian is negative semi-definite. Using the expressions in Proposition 4, we can derive the lower bound for $d^2\eta / d\lambda^2$ at $\lambda = \lambda^*$. The upper bound is guaranteed to be larger than the lower bound by Lemma 1. To ensure that $\eta(\lambda)$ provides unique maximum, we further require the concavity bounds to hold for all $\lambda$. Part iii is necessary to guarantee both marketing and IT earn positive profits.

Proof of Proposition 6

With the Lagrange multiplier $\xi_D$, we write the first order condition,

$$h \left( 1 - \frac{\nu(\mu + \nu)}{(\mu - \lambda + \nu)^2} \cdot (1 + x_i\sigma^2) \right) = \xi_D - 2(\mu + \nu)x_i\sigma^2;$$

$$h \frac{\nu\lambda}{(\mu - \lambda + \nu)^2} \cdot (1 + x_i\sigma^2) = \gamma + \lambda x_i\sigma^2;$$

where the two parameters,
\[ x_1 = \frac{3(\mu + \nu)}{(\mu - \lambda + \nu)^2} \sigma^2; \text{ and, } x_2 = \frac{hv}{(\mu - \lambda + \nu)^3} \sigma^2. \]

Since \( \sigma^2 \) is small, we find solutions up to the terms linear in \( \sigma^2 \),

\[ \lambda_{D,IT} = \frac{\nu h \gamma_1}{(h - \xi_D - \gamma_1)^2 + (\mu' - \lambda' + \nu)^2 \sigma^2}; \]

\[ \mu_D = \frac{\nu h (h - \xi_D)}{(h - \xi_D - \gamma_1)^2} - \nu + \frac{\mu' + \nu}{(\mu' - \lambda' + \nu)^2} \sigma^2. \] (A4)

Similarly \( \xi_D \) can be obtained by setting \( \lambda_{D,IT} = \lambda_{D,M} \). \( \lambda_{D,M} \) has the same form of \( \lambda_M \), however with \( \xi \) replaced by \( \xi_D \). This yields,

\[ \Delta \xi = \xi_D - \xi^* = -\left( \frac{\partial g(\xi)}{\partial \xi} \right)^{-1} \frac{\lambda^*}{(\mu^* - \lambda^* + \nu)^2} \sigma^2 < 0, \]

and,

\[ \Delta \lambda = \lambda_D - \lambda^* = \frac{\partial \lambda_{D,M}}{\partial \xi} \Delta \xi < 0. \]

The sign of \( \Delta \mu = \mu_D - \mu^* \) is determined by Equation A4. After some manipulations, we have,

\[ \Delta \mu \propto \frac{\nu h \gamma_1}{(h - \xi^* - \gamma_1)^2} - \frac{(h - \xi^*)a}{\xi^*} \left( 1 - \frac{4a}{\lambda_M \xi^*} \right)^{-1/2}. \]

The first term is nothing but \( \lambda^* \) which decreases with \( \gamma_1 \). The second term is a decreasing function of \( \xi^* \). As \( \xi^* \) decreases with \( \gamma_1 \), the second term increases with \( \gamma_1 \). Therefore the sign of \( \Delta \mu \) may change from positive to negative as \( \gamma_1 \) is increased beyond a threshold. To find this threshold, we set \( \Delta \mu = 0 \) and simultaneously solve \( \xi^* \) by \( g(\xi^*) = 0 \). This leads to Proposition 6.

**PROOF OF PROPOSITION 7**

The first order condition is,

\[ h \left( 1 - \frac{1}{2} \frac{\nu(\mu + \nu)}{(\mu - \lambda + s + \nu)^2} - \frac{1}{2} \frac{\nu(\mu + \nu)}{(\mu - \lambda - s + \nu)^2} \right) - \xi_{NS} = 0; \]

\[ \frac{hv}{2} \left( \frac{\lambda - s}{(\mu - \lambda + s + \nu)^2} + \frac{\lambda + s}{(\mu - \lambda - s + \nu)^2} \right) - \gamma_1 = 0. \]

This yields the capacity and demand,

\[ \mu_{NS} = V \frac{h \xi_{NS}}{(h - \xi_{NS} - \gamma_1)^2} \left( 1 + \frac{y}{2\sqrt{y^2 + s^2}} \right) - V; \]

\[ \lambda_{NS,IT} = V \frac{h \gamma_1}{(h - \xi_{NS} - \gamma_1)^2} \left( 1 + \frac{y}{2\sqrt{y^2 + s^2}} \right) - \frac{s^2}{\sqrt{y^2 + s^2}}, \]

where the parameter,

\[ y = \frac{hv}{2(h - \xi_{NS} - \gamma_1)}. \]
The expression for $\lambda_M$ remains unchanged. Notice that if $s > 0$, for the same value of $\xi$, $\lambda_{NS, IT} < \lambda_{IT}$. This indicates that when equating $\lambda_{NS, IT}$ to $\lambda_M$ to obtain $\xi_{NS}$, it is required that $\xi_{NS} > \xi^*$. Consequently, we find that $\lambda_{NS} > \lambda^*$. To show that $\mu_{NS} > \mu^*$, we rewrite,

$$\mu_{NS} = \lambda_{NS, IT} - \nu + y + \sqrt{y^2 + s^2},$$

where $y$ is an increasing function of $\xi_{NS}$.

**2. Short Term Problem**

In the following, we solve the short term problem where the capacity is fixed at $\mu = \mu_0$.

**2.1 Centralized Solution**

The profit function for the centralized setting is given as

$$\pi_s = h(\lambda - L(\lambda, \mu)) - A = h\left(\lambda - \nu \frac{\lambda}{\mu_0 - \lambda + \nu}\right) - A.$$

The optimal demand is obtained by solving,

$$R_s(\lambda) \equiv h\left(1 - \nu \frac{\mu_0 + \nu}{(\mu_0 - \lambda + \nu)^2}\right) - \frac{a\lambda_s}{\lambda(\lambda_0 - \lambda)} = 0.$$

It has two roots; the larger root satisfies $dR_s/d\lambda \leq 0$ (concavity) and hence is the local interior maximum. Different from the long-term problem, when the advertising cost parameter $a$ is very high, this maximum may fall below $\lambda_0/2$ but the boundary maximum at $\lambda_0$ is typically superior. Therefore, in the following, we assume that the advertising cost parameter $a$ is in a range such that the optimal demand is above $\lambda_0/2$.

Similarly, we can rewrite the solution in an alternative form. Introduce a real number $\xi_s$. The optimal demand can be found by solving the balance equation, $\lambda^* = \lambda_{IT}(\xi_s) = \lambda_M(\xi_s)$, where,

$$\lambda_{IT}(\xi_s) = \mu_0 + \nu - \frac{h(\mu_0 + \nu)}{h - \xi_s}; \quad \text{and} \quad \lambda_M(\xi_s) = \frac{\lambda_0}{2} \left(1 + \sqrt{1 - \frac{4a}{\xi_s \lambda_0}}\right).$$

**2.2 Reduced Session Value**

In the above formulation, coordination is still necessary. If marketing ignores lost sales due to customer impatience, it will over-advertise and choose a sub-optimal demand level. As in the long-run problem, information about customer’s impatience can be aggregated in $\xi_s^*$ and conveyed to marketing. Normalized $\xi_s^* / h$ is a function of three (composite) parameters, $4a/h\lambda_0$, $\mu_0/\nu$, and $\lambda_0/2\nu$.

Table 1 shows how the reduced per session value varies with the advertising cost and IT capacity. For the short term problem, the flexibility of optimally adjusting the IT capacity is lost. As a result, $\xi_s^*$ is more sensitive to advertising costs when the capacity is low. A higher level of capacity significantly reduces the dependence of $\xi_s^*$ on the advertising cost.

From Table 2, it is apparent that $\xi_s^*$ can still be used to effectively aggregate IT’s operational details. IT can either under- or over-estimate the advertising cost when calculating $\xi_s^*$. Table 2 shows the reduction in profit when a false value of $a$ is used (the true value is 0.8). Overall, the reduction is small; however, over-estimation is a better option.
Table 1. Normalized reduced per session value $\xi_s^*/h$ for $\lambda_\infty/\nu = 10$

<table>
<thead>
<tr>
<th>$4a/h\lambda_\infty$</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
</tr>
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<td>0.4000</td>
<td>0.6531</td>
<td>0.7654</td>
<td>0.8264</td>
<td>0.8639</td>
<td>0.8889</td>
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<td>0.9197</td>
</tr>
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<td>0.7818</td>
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</tr>
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<td>0.8448</td>
<td>0.8758</td>
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</tr>
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<td>0.7434</td>
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<td>0.9157</td>
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</tr>
<tr>
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<td>0.7670</td>
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<td>0.8621</td>
<td>0.8874</td>
<td>0.9055</td>
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<td>0.9291</td>
</tr>
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<td>0.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
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</table>

Table 2. Impact of inaccurate advertising cost information ($\mu_0/\nu = 14$)

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\xi_s^*$</th>
<th>$\hat{\lambda}$</th>
<th>PROFIT REDUCTION</th>
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</thead>
<tbody>
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<td>0.4000</td>
<td>7.2362</td>
<td>3.7764 6.69%</td>
</tr>
<tr>
<td>0.1</td>
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<td>7.6945</td>
<td>3.9193 3.16%</td>
</tr>
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<td>3.9799 1.66%</td>
</tr>
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<td>4.0111 0.89%</td>
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<td>8.4168</td>
<td>4.0439 0.08%</td>
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<td>0.7369</td>
<td>8.7608</td>
<td>4.0342 0.32%</td>
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</table>

2.3 Processing Contract for Coordination

Similarly we can construct a processing contract, $\eta(\lambda)$, so that the net value maximizing demand can be induced. The profit for IT is $\pi^*_{IT} = \eta(\hat{\lambda}) - hL(\hat{\lambda}, \mu_0)$. Different from the long term problem, here, a simple linear form $\eta(\lambda) = \eta_0 + (h - \xi_s^*)\lambda$ would coordinate. This is due to the fact that the loss function $L$ is convex in $\lambda$. Next, we examine whether profit sharing is necessary. Setting $\eta_0 = 0$, at the optimal demand level, the profit for IT is,

$$\pi^*_{IT} = \left(\sqrt{(h - \xi_s^*)(\mu_0 + \nu)} - \sqrt{hv}\right)^2 > 0.$$  

This indicates that IT can be operated as a profit center even in the absence of profit sharing.