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APPENDIX A. PROOF OF LEMMAS AND PROPOSITIONS

PROOF OF LEMMA 1.

(a) Observe that $W_S^*(Q)$ is the maximum over a set of functions linear in Q, and is therefore convex (Rockafellar 1970).

(b) $W_S^*(Q) = U_S(N_S)Q-K_S(N_S) \geq (U_S(N_S^*)Q-K_S(N_S^*)$, from the definition of $N_S(Q)$.

(C) It is sufficient to show that $N_S(Q) \rightarrow N_S^*$ for large Q. The first order condition for the self-service problem may be written as $Q\left[\frac{fU_S(N)}{fN} - \frac{K_S'(N)}{Q}\right] = 0$. $K_s'(N)$ is bounded above since $N_S(Q)$ is. Thus, the solution to the above first order condition approaches the solution to $Q\frac{fU_s(N)}{fN} = 0.$

PROOF OF LEMMA 2. It is easy to see this result by manipulating the benefit functions in Figure 3.

PROOF OF LEMMA 3. The breakeven volumes satisfy $W_S^*(Q) = U_p(N_p)Q-P_p$. For (a), differentiate both sides with respect to P_p giving, $\partial Q/\partial P_p = 1/(U_p-\partial W_S^*/\partial Q)$. Note that at Q_{p1}, $U_p-\partial W_S^*/\partial Q > 0$, while $U_p-\partial W_S^*/\partial Q < 0$ at Q_{p2}, giving the first part of (a). The sign of the second derivatives follow from differentiating again with respect to the fixed charge, resulting in $(\partial Q/\partial P_p)^2\partial^2 W_S^*/\partial Q^2 = (U_p-\partial W_S^*/\partial Q)\ \partial^2 Q/\partial P_p^2$. The proof of (b) is very similar. The cross derivative of the breakeven volume is given by:

$\partial Q/\partial N_p \partial Q/\partial P_p \partial^2 W_S^*/\partial Q^2 - U_p' \partial Q/\partial P_p = (U_p-\partial W_S^*/\partial Q)\ \partial^2 Q/\partial N_p \partial P_p$, whereby (c) follows.

PROOF OF PROPOSITION 1 AND COROLLARY 1. Differentiate $mP_p(Q_{p2}-Q_{p1})/Q_{max}$ with respect to P_p, giving the first order condition for the revenue maximizing P_p as $Q_{p2}-Q_{p1} + P_p\partial(Q_{p2}-Q_{p1})/\partial P_p = 0$. The second order condition can be expressed as:

$\partial^2 REV_p(N_p,P_p)/\partial P_p^2 = m\left(2\partial(Q_{p2}-Q_{p1})/\partial P_p + P_p\partial^2 (Q_{p2}-Q_{p1})/\partial P_p^2\right)/Q_{max} < 0$, from Lemma 3(a). This establishes the concavity of the revenue function and the uniqueness of $P_p^{mon}(N_p)$. The cross-derivative of the first order condition with respect to the database size may be written as:

$\partial^2 REV_p(N_p,P_p)/\partial N_p\partial P_p=\partial(Q_{p2}-Q_{p1})/\partial N_p+dP_p/dN_p\partial(Q_{p2}-Q_{p1})/\partial P_p+P_p\partial^2(Q_{p2}-Q_{p1})/\partial N_p\partial P_p$

If $N_p > U_p^{-1}(U_s^*)$, then $Q_{p2} = Q_{max}$, whereby the above expression reduces to:

$-dP_p/dN_p\partial Q_{p1}/\partial P_p = \partial Q_{p1}/\partial N_p + P_p\partial^2 Q_{p1}/\partial N_p\partial P_p < 0$ implying $dP_p^{mon}(N_p)/dN_p > 0$.

Corollary 1 follows from the monotonicity of $P_p^{mon}(N_p)$.

PROOF OF PROPOSITION 2. Differentiate $mP_p(N_p)(Q_{p2}-Q_{p1})/Q_{max}$ with respect to N_p, giving slope of the revenue function at the optimal fixed charge as:

$$m\left((Q_{p2}-Q_{p1})dP_p/dN_p + P_p(N_p)\partial(Q_{p2}-Q_{p1})/\partial N_p\right)Q_{max} > 0$$

from Proposition 1 and Lemma 3(b). Thus, the optimal revenues are monotonically increasing in the database size. If $K_p(0) \geq REV_p(N_p^b)$, then the optimal database size will be greater than N_p^b. The result in (b) is obvious since once the fixed costs are incurred, the monopolist would create the largest feasible database, if there are no marginal costs. Let $\varepsilon_i = \partial W_s^*/\partial Q\big|Q=Q_{pi}$ be the slope of the self service benefit function at the breakeven volume Q_{pi}, i=1,2. Also let $K_{\varepsilon_i} = K_s(U_s^{-1}(\varepsilon_i))$ be the cost of building a self service database by a subscriber of size Q_{pi}. Then, $Q_{p1} = (P_p-K_{\varepsilon_1})/(U_p-\varepsilon_1)$, whereby the revenue maximizing fixed charge may be expressed as:

$$P_p(N_p) = \left((U_p-\varepsilon_1)Q_{max} + K_{\varepsilon_i}\right)/2,$$

thus, the first order condition for the profit maximizing database size can be written as:

$mU_p'\left(Q_{max}^2-K\varepsilon_i^2/(U_p-\varepsilon_1)^2\right)/4Q_{max} - K_p'(N_p) = 0$. If the database building cost is linear as indicated, then, the profit maximizing database size is given by:

$dU_p/dN_p = 4c_pQ_{max}/\left(m(Q_{max}^2-K\varepsilon_i^2/(U_p-\varepsilon_1)^2\right) \geq 4c_p/mQ_{max}$ since $K\varepsilon_1 \geq 0$. Since U_p is concave, the above inequality implies that $N_p^{mon} \leq U_p'^{-1}(4c_p/mQ_{max})$.

PROOF OF LEMMA 4. Similar to Lemma 3.

PROOF OF LEMMA 5. The first order condition for the marginal price in the pricing sub-problem may be written as: $\partial REV_o/\partial p_o = \partial REV_o^{sub}/\partial p_o + \partial REV_o^{use}/\partial p_o$

It is straightforward to show that the first term is always negative. The second term can be expressed as: $\partial REV_o^{use}/\partial p_o = m\left(2(Q_{o2}\partial Q_{o2}/\partial p_o - Q_{o1}\partial Q_{o1}/\partial p_o) + (Q_{o2}^2-Q_{o1}^2)\right)/2Q_{max}$

Note that $\partial Q_{oi}/\partial p_o = Q_{oi}/(U_o-p_o-\varepsilon_i)$, where $\varepsilon_i = \partial W_s^*/\partial Q\big|Q=Q_{oi}$ is the slope of the self service benefit function at the breakeven volume Q_{oi}, i=1,2. Also, $\varepsilon_i \leq U_s^*$ from Lemma 1. Making these substitutions in the above, we get

$$\frac{\partial REV_o^{use}}{\partial p_o} = Q_{o2}^2\sqrt{\frac{U_o+p_o-\varepsilon_2}{U_o-p_o-\varepsilon_2}} - Q_{o1}^2\sqrt{\frac{U_o+p_o-\varepsilon_1}{U_o-p_o-\varepsilon_1}}.$$ Note that $U_o- p_o < \varepsilon_2$, by definition of Q_{o2}. The above derivative is negative if $Q_{o2} < Q_{max}$, which condition is attained whenever $p_o > U_o- U_s^*$.

PROOF OF PROPOSITION 3. The proof of (a) is similar to Proposition 2. If $N_o \geq U_o^{-1}(U_s^*)$, then the optimal fixed charge solves: $Q_{max}-Q_{o1} -(P_o+ p_oQ_{o1})\partial Q_{o1}/\partial P_o = 0$, where $\partial Q_{o1}/\partial P_o = 1/(U_o-p_o-\varepsilon_1)$. From Lemma 5, $\partial REV_o/\partial p_o = m\left(-(P_o+p_oQ_{o1})\partial Q_{o1}/\partial p_o + (Q_{max}^2-\right.$

$Q_{o1}^2)/2)/Q_{max}$ where $\partial Q_{o1}/\partial p_o = Q_{o1}/(U_o\text{-}p_o\text{-}\varepsilon_1)$. Substituting for the optimal fixed charge and the other derivatives in above equation, we get $\partial REV_o/\partial p_o = m(Q_{max}\text{-}Q_{o1})^2/2Q_{max} > 0$, establishing (b). Thus, the optimal fixed marginal charge is $p_o^{mon} = U_o\text{-}\ U_s^*$, giving (c). For (d), differentiate the first order condition for the optimal fixed charge with respect to N_o, substitute $p_o = U_o\text{-}\ U_s^*$, and rearrange the terms to obtain:

$$\frac{dP_o^{mon}}{dN_o} = -(Q_{max} - Q_{o1})\frac{f^2 W_s^*}{fQ_{o1}^2}\frac{fQ_{o1}}{fN_o} - Q_{o1}\frac{dp_o^{mon}}{dN_o} - \frac{fQ_{o1}}{fN_o}(p_o^{mon} + U_s^* - \varepsilon_1)\,.$$

Note that $\partial Q_{o1}/\partial N_o = -Q_{o1}U_o'/(U_o\text{-}p_o\text{-}\varepsilon_1) = -Q_{o1}U_o'/(U_s^*\text{-}\varepsilon_1)$, and $dp_o^{mon}/dp_o = U_o'$. Further substituting these in the above equation results in:

$$\frac{dP_o^{mon}}{dN_o} = -(Q_{max} - Q_{o1})\frac{f^2 W_s^*}{fQ_{o1}^2}\frac{fQ_{o1}}{fN_o} + U_o' Q_{o1}\frac{U_o - U_s^*}{U_s^* - \varepsilon_1} > 0$$

since the first term is positive because of Lemma 1, while $U_o' > 0$ because $N_o \le N_o^*$.

Finally, $P_o^{mon} = (Q_{max}\text{-}Q_{o1})(U_s^*\text{-}\varepsilon_1)\text{-}p_o^{mon}Q_{o1} < (Q_{max}\text{-}P_o^{mon}/U_s^*)\text{-}p_o^{mon}P_o^{mon}/U_s^*$ since $Q_{o1} > P_o^{mon}/U_s^*$ and $\varepsilon_1 \ge 0$. The upper bound for the optimal fixed price follows on rearranging the terms. Note that as in Proposition 2, Q_{o1} may be written as $Q_{o1} = (P_o^{mon}\text{-}K_{\varepsilon_1})/(U_s^*\text{-}\varepsilon_1)$, which on substitution in the formula for the optimal fixed charge results in:

$$P_o^{mon} = \frac{Q_{max}(U_s^* - \varepsilon_1)^2 + K_{\varepsilon_1}(U_o - \varepsilon_1)}{U_o + U_s^* - 2\varepsilon_1}$$

which is equal to zero when $K_s(N_s) \equiv 0$ because $\varepsilon_1 = U_s^*$ and $K_{\varepsilon_1} = 0$.

PROOF OF PROPOSITION 4. The derivative of the revenue function for N_o can be written as:

$$\frac{dREV_o(N_o)}{dN_o} = \frac{m}{Q_{max}} \left((Q_{o2} - Q_{o1}) \frac{dP_o}{dN_o} + P_o^{mon} \frac{f(Q_{o2} - Q_{o1})}{fN_o} + p_o^{mon} \left(Q_{o2} \frac{fQ_{o2}}{fN_o} - Q_{o1} \frac{fQ_{o1}}{fN_o} \right) + (Q_{o2}^2 - Q_{o1}^2) \frac{dp_o^{mon}}{dN_o} \right)$$ which is

clearly positive because the derivatives of the fixed and marginal prices are positive from Proposition 3, while Lemma 4(b) ensures the positivity of the remaining terms. With this observation, (a) and (b) follow in a manner similar to Proposition 2. The bound in (c) corresponds to the case where $K_S(N_S) \equiv 0$, consequent from a derivation similar to Proposition 2(c), where the required the marginal and fixed prices are derived in Proposition 3(e).

PROOF OF PROPOSITION 5.

LEMMA. If both providers survive, they will employ different database sizes and prices, such that $P_{p1} > P_{p2}$ and $N_{p1} > N_{p2}$, and make identical profits.

PROOF OF LEMMA. Assume that if both suppliers use identical product design and pricing strategies, they split the market equally. Suppose both suppliers survive in the market, but $N_{p1} = N_{p2} = N$, $P_{p1} = P_{p2} = P$ in equilibrium. Keeping a database size of N, Firm 2 can then price at a level infinitesimally smaller than firm 1, and capture the entire market. Firm 1 reacts likewise, and the two firms undercut each other till one firm charges the smallest price $P_0(N)$ such that $mP_0(N)MS(N,P_0(N))- K_{pi}(N) = 0$. The other firm then stays out of the market, which reverts to a monopoly. A similar argument applies when both attempt to change their product designs while charging identical prices in equilibrium. Thus, if both firms survive, they utilize different pricing and design strategies. Clearly, the higher priced firm will then use a larger database,

otherwise, it will be dominated. In addition, they must make identical profits, otherwise, the lower profit firm can mimic the size of the other firm, and undercut its price and do better.

PROOF OF PROPOSITION. Suppose the equilibrium described in the lemma exists. Without loss of generality, assume that both firms make zero profits. Then, it must be that $P_{p1} > P_{p2} > P_0(N_{p2})$, since $P_0(N_{a2})$ is the smallest price at which either firm makes a zero profit as a monopolist with a size N_{p2}. Then, Firm 1 can mimic Firm 2's size, undercut Firm 2's price, and do better. When this happens, firms engage in intense price and size competition till one firm charges $P_0(N_p^*)$, while the other firm stays out, reverting the market to monopoly.

PROOF OF LEMMA 6. Similar to Lemma 3.

PROOF OF LEMMA 7. The first order condition for the marginal price in the pricing sub-problem may be written as: $\partial REV_o/\partial p_o = \partial REV_o^{sub}/\partial p_o + \partial REV_o^{use}/\partial p_o$

It is straightforward to show that the first term is always negative. The second term can be expressed as: $\partial REV_o^{use}/\partial p_o = m\big(2(Q_{po}\partial Q_{po}/\partial p_o - Q_{o1}\partial Q_{o1}/\partial p_o) + (Q_{po}^2 - Q_{o1}^2)\big)/2Q_{max}$

Note that $\partial Q_{o1}/\partial p_o = Q_{o1}/(U_o - p_o - \varepsilon_1)$, where $\varepsilon_1 = \partial W_s^*/\partial Q\big|Q=Q_{o1}$ is the slope of the self service benefit function at the breakeven volume Q_{o1}. Also, $\varepsilon_1 \le U_s^*$ from Lemma 1. Making these substitutions in the above, we get

$$\frac{\partial REV_o^{use}}{\partial p_o} = Q_{po}^2 \frac{U_p - U_o - p_o}{U_p - U_o + p_o} - Q_{o1}^2 \frac{U_o + p_o - \varepsilon_1}{U_o - p_o - \varepsilon_1}.$$ The above derivative is negative if $p_o > U_p - U_o$.

PROOF OF LEMMA 8. For any N_o, the equilibrium fixed charge in this case solves the first order condition: $Q_{po}-Q_{o1}+(P_o+p_oQ_{po})\partial Q_{po}/\partial P_o-(P_o+p_oQ_{o1})\partial Q_{o1}/\partial P_o = 0$, where $\partial Q_{o1}/\partial P_o = 1/(U_o-p_o-\varepsilon_1)$. The corresponding derivative for the marginal charge can be written as:

$$\partial REV_o/\partial p_o = m\left((P_o+p_oQ_{po})\partial Q_{po}/\partial p_o-(P_o+p_oQ_{o1})\partial Q_{o1}/\partial p_o+(Q_{po}^2-Q_{o1}^2)/2\right)/Q_{max}$$

Substituting the formula for the equilibrium fixed charge in the above gives the following expression for the above derivative,

$$\frac{fREV_o}{fp_o} = -\frac{(Q_{po}-Q_{o1})^2(U_p-2U_o+\varepsilon_1)}{2(\delta_{po}-\delta_{o1})(U_p-U_o+p_o)(U_o-p_o-\varepsilon_1)}$$

where $\delta_{ab} = \frac{1}{Q_{ab}}\frac{fQ_{ab}}{fp_b} < 0$ and $\delta_{b1} = \frac{1}{Q_{b1}}\frac{fQ_{b1}}{fp_b} > 0$. Note that the feasible set of marginal charges is given by the set $[0,\min(U_o-U_s^*,U_p-U_o)]$. The lemma follows by evaluating the derivative in the three intervals indicated therein.

PROOF OF LEMMA 9. For any N_o, the equilibrium fixed charge in this case solves the first order condition: $Q_{po}-Q_{o1}+(P_o+p_oQ_{po})\partial Q_{po}/\partial P_o-(P_o+p_oQ_{o1})\partial Q_{o1}/\partial P_o = 0$,

where $\partial Q_{o1}/\partial P_o = 1/(U_o-p_o-\varepsilon_1)$. On substituting out the derivatives, the above formula simplifies to:

$$Q_{po}\sqrt[3/4]{\frac{U_p-U_o}{U_p-U_o+p_o^{duo}}}-Q_{o1}\sqrt[3/4]{\frac{U_o-\varepsilon_1}{U_o-\varepsilon_1-p_o^{duo}}}-P_o\sqrt[3/4]{\frac{U_o-\varepsilon_1}{(U_o-\varepsilon_1-p_o^{duo})(U_p-U_o+p_o^{duo})}} = 0$$

In the indicated ranges in the Lemma, substitute the equilibrium marginal charge from Lemma 8, and the breakeven volumes as functions of P_o, and rearrange the terms to get:

$$p_o^{duo} = \begin{cases} \dfrac{P_p(U_o - \varepsilon_1) + K\varepsilon_1(U_p - U_o)}{2(U_p - \varepsilon_1)} & U_o > \dfrac{U_p + \varepsilon_1}{2} \\ \dfrac{P_o(U_s^* - \varepsilon_1)^2(U_p - U_o) + K\varepsilon_1(U_o - \varepsilon_1)(U_p - U_s^*)^2}{(U_s^* - \varepsilon_1)^2(U_p - U_o) + (U_o - \varepsilon_1)(U_p - U_s^*)^2 + (U_p - U_s^*)(U_s^* - \varepsilon_1)(U_p - \varepsilon_1)} & U_s^* \le U_o \le \dfrac{U_p + \varepsilon_1}{2} \end{cases}$$

Finally, substitute the response of the package provider given below to complete the Lemma:

$$P_p = \frac{P_o + Q_{max}(U_p - U_o + p_o)}{2}$$

Part (c) follows by using the fact that $Q_{o1} > P_o^{duo}/U_s^*$. Details are omitted.

PROOF OF LEMMA 10. Outline is provided in the text.

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