Online Companion for

"Designing Portfolios of Financial Products via Integrated Simulation and Optimization Models"

Operations Research
Vol. 47, No. 2, March-April 1999

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Designing Portfolios of Financial Products via Integrated Simulation and Optimization Models

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July 1996.
Revised, February 1998.

A Scenarios of Holding Period Returns for Callable Bonds

In this Appendix we give some background information on callable bonds, and discuss the simulation procedures for computing scenarios of holding-period returns for these bonds.

The holder of a callable bond gives the issuer the right to redeem the bond before its maturity date. Corporations, municipalities, utility companies and the National mortgage agencies issue callable bonds as debt-raising instruments. The ability to refinance their debt in a potentially lower-interstate environment is extremely valuable to the issuers. However, earlier-than-expected retirement of a bond with higher-than-market coupon rate poses a risk to investors. Hence, callable bonds are usually traded at a premium above non-callable bonds. For high-quality issuers this premium is typically 30 to 40 bp. Typically there is a 5 to 10-year wait period before a bond can be called. After this lockout period the bonds can be called at the option of the issuer at any time, with call prices that are set at a premium above par and decline linearly on an annual basis to par, 5 to 10 years before final scheduled maturity.

At the portfolio level callable bonds went out of fashion in the 1990s. As interest rates declined investors saw a sizable portion of their callable bond portfolios liquidated. Rating agencies warned U.S. insurers about their excessive exposure to call risk, and several insurers diversified their portfolios away from bonds with call (or embedded-call) options. Nevertheless, callable bonds remain a preferred debt-issuing instrument for utilities and the mortgage agencies. Although today they trail, in performance, their non-callable counterparts, the callable bond sector will eventually rebound if interest rates remain static or reverse.

Further background information on callable bonds can be found on the Web sites http://www.encarta.msn.com/ and http://www.dbcinc.com/.

A.F.Best Securities Inc. http://www.afbest.com/ and First Institutional Securities, LLC, http://www.firstinstitutional.com/ provide on-line advice on municipal bonds, including several callable issues. The books by Zenios (1993) and Censor and Zenios (1997, ch. 14) give models for risk-management with fixed-income securities, including corporate callable bonds; see http://www.cup.cam.ac.uk/ and http://www.oup.co.uk/, respectively.

We discuss now with the generation of scenarios of holding-period returns for callable bonds. We start with a model for pricing a callable bond. Once a pricing model is developed, the generation of scenarios of holding period returns is straightforward. All calculations in this section are given for a generic bond, and we drop the dependence of the holding period returns on the bond index j or the design parameters p^j.

A.1 Pricing a callable bond

The value of a callable bond depends on three factors: The term structure of the Treasury (i.e., risk free) interest rates, the credit risk of the issuing institution, and the value of the embedded option.

A.1.1 The term structure model

Risk free interest rates are generated by the binomial lattice model of Black, Derman and Toy (1990). This model generates distributions of interest rates that are arbitrage free, and that price correctly the points on the spot yield curve. The volatility of these rates is also consistent with the term structure of volatility. The binomial lattice can be described as a series of *base* rates $\{r_t^0,\ t = 0, 1, 2, \ldots, T\}$ and *volatilities* $\{k_t,\ t = 0, 1, 2, \ldots, T\}$, where T denotes the end of the planning horizon (typically 360 months). The short-term rate at any state σ of the binomial lattice at some time instance t is given by $r_t^\sigma = r_t^0 (k_t)^\sigma$.

The price of a future stream of risk free cashflows — such as those generated, for example, by a U.S. Treasury security — can be obtained as the net present value of these cashflows, with discounting done at the rates r_t^σ.

A.1.2 Credit risk

The cashflows generated by a callable bond can not be priced simply by discounting them at the risk free rate. Market participants assume the default, liquidity and other risks inherent in callable bonds issued by corporations, utilities or government agencies. The discount rates should reflect these risks. Therefore the risk free rates must be adjusted by appropriate *risk premia*. Of primary interest to an institution issuing callable bonds is the credit risk premium. A model for the term structure of credit risk has been developed by Litterman and Iben (1991). In particular they show how to

obtain a vector of risk premia $\{\rho_t\}$ that are used to adjust the risk free rates so that the discounted cashflows of non-callable corporate bonds are priced correctly.

The term structure of the credit risk for an institution is obtained using the Litterman-Iben model from existing traded securities issued by the institution. These risk premia are applied to the pricing of new issues of callable bonds.

A.1.3 Value of the option and of the callable bond

We can now value the bond. A callable security can be viewed as a long position in a non-callable bond, and a short position in a call option on that underlying bond. The price of the callable security is the difference between the non-callable bond value and the option premium.

The non-callable bond can be priced starting at the end of the binomial lattice — when the security is priced at par — discounting its price backwards, and computing the average value. Figure 1 illustrates the procedure on a simple binomial lattice. If $P_{NC}^{t,\sigma}$ denotes the price of the non-callable bond at time period t and state σ (with risk-free discount rate $r_t^\sigma = r_t^0(k_t)^\sigma$), then $P_{NC}^{t,\sigma}$ can be computed by applying the recursive equation

$$P_{NC}^{t,\sigma} = \frac{1}{2}\frac{P_{NC}^{t+1,\sigma} + P_{NC}^{t+1,\sigma+1}}{1 + r_t^\sigma \rho_t}, \tag{1}$$

where $P_{NC}^{T,\sigma} = 100$ (par) for all states σ at maturity T.

The value of the call option is calculated using the same discounting method used above for the non-callable bond. After the lockout period — when the call may be exercised — its value can not fall below the difference between the non-callable bond and the redemption price; see Figure 1. If $P_O^{t,\sigma}$ denotes the price of the call option at time period t and state σ, and R^t denotes the redemption price, then $P_O^{t,\sigma}$ can be computed by applying the recursive equation:

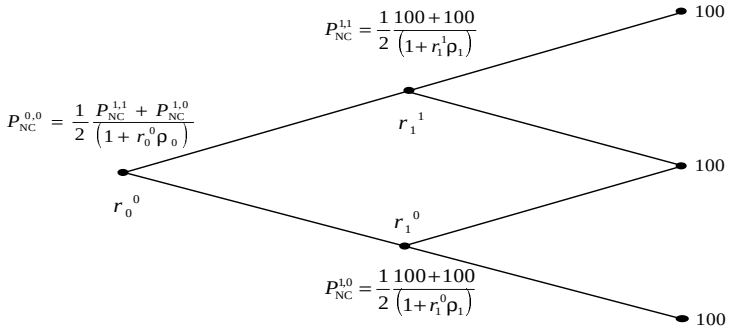
$$P_O^{t,\sigma} = \max\left\{\frac{1}{2}\frac{P_O^{t+1,\sigma} + P_O^{t+1,\sigma+1}}{1 + r_t^\sigma \rho_t}, (P_{NC}^{t,\sigma} - R_t)\right\}. \tag{2}$$

Note that the redemption price is infinite before the lockout period expires, and is zero at maturity.

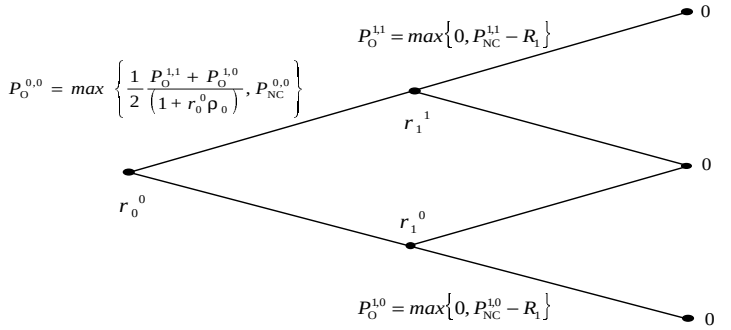
Finally, the value of the callable bond $P_C^{t,\sigma}$, at period t and state σ, is given as the difference between the non-callable bond price and the value of the call option. That is,

$$P_C^{t,\sigma} = P_{NC}^{t,\sigma} - P_O^{t,\sigma}. \tag{3}$$

(1) Pricing a non-callable bond.



(2) Pricing a call option, callable at R_1 in one year and at $R_2 = 100$ in two years.



(3) Subtracting the values of the call option from the price of the non-callable bond at each node of the tree we get the price of the callable bonds.

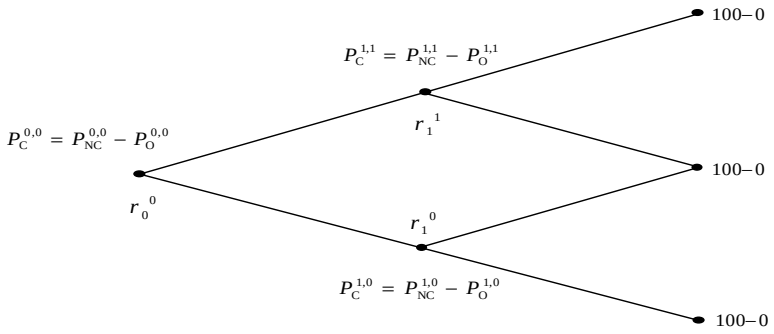


Figure 1: Pricing a non-callable bond, a call option and a callable bond using a binomial lattice.

A.2 Holding period returns

The return of a callable bond during a holding period (τ), under an interest rate scenario s, is given by

$$r^s = \frac{P^s + F^s}{P_0}. \tag{4}$$

Here P^s is the value of the bond at the end of the holding period (assuming it has not been called), F^s is the accrued value of any coupon payments made during the holding period *plus* the accrued value of the cashflow generated if the bond were called, and P_0 is the current market price. In order to evaluate the holding period return we need to specify the interest rate scenario (s) that was realized during the holding period. This scenario will determine whether the bond is called or not during the holding period.

To generate scenarios of holding period returns we sample *paths* from the binomial lattice starting at the origin $t = 0$ and ending at $t = \tau$. Each path corresponds to a scenario s. We then traverse the path on the binomial lattice, check whether the call option has been exercised or not, and compute the value of F^s taking also into account any coupon payments. (All relevant information has already been generated during the pricing calculations described above). The price of the bond at the end of the holding period under the given scenario, P^s, is also available from the pricing calculations.

References

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