Collaborative Transportation with Optional Customers

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1. Introduction
Collaborative transportation among carriers has the potential to greatly increase the efficiency of their transportation networks. By collaborating, the carriers can significantly reduce their cost (Guajardo and Rönqvist 2015), increase their service levels and improve their market share (Gansterer and Hartl 2018). Moreover, such collaborations are known to provide numerous other benefits including reductions in noise pollution, road congestion and green house gas emissions as shown by Ballot and Fontane (2010) and Pérez-Bernabeu et al. (2014).

In this paper, we study a horizontal collaboration among multiple carriers, each responsible for multiple customers with a given demand. It is assumed that each carrier serves the demand of its customers using a limited fleet of capacitated vehicles from a depot. Moreover, we assume that the carriers are rational and aim to maximise their profits. The carriers may collaborate by consolidating demands and combining delivery routes. As a consequence, the carriers are likely to have residual capacity. This residual capacity can be used by the carriers to generate additional profits by collectively serving new customers. We focus on the profit allocation problem that arises when multiple carriers collaborate by consolidating demands, combining delivery routes and collectively serving new customers.

Profit, or equivalently, cost allocation problems in collaborative transportation are often modelled as cooperative games (Guajardo and Rönqvist 2015). A cooperative game is defined as a pair consisting of a set of players, also known as the grand-coalition, and a characteristic function which maps each subset of players, also referred to as a coalition, to a value which we interpret as the profit of a coalition. We aim to allocate the profit of the grand-coalition to the players such that each player has an incentive to cooperate. To this end, it is common to consider core allocations
van Zon and Desaulniers: Collaborative Transportation with Optional Customers

(Guajardo and Rönnqvist 2015). A core allocation is an allocation such that the profit allocated to each coalition exceeds the profit which the coalition would be able to obtain without any form of collaboration with the remaining players of the grand-coalition.

Originally, collaborative transportation was studied from the perspective of the customers, rather than the carriers. For example, Engevall, Göthe-Lundgren, and Värbrand (2004) consider a setting in which multiple customers are served by a single carrier. More recently, collaborative transportation has also been studied from the perspective of carriers. For example, Krajewska et al. (2008) and van Zon, Spliet, and van den Heuvel (2019) study a profit allocation problem which arises from a collaboration among freight carriers. The previously mentioned studies are mainly concerned with optimising the efficiency of the existing routing networks through collaborative transportation, but do not consider any additional gains which could be obtained as a consequence of the collaboration.

We define a new cooperative game where we do consider the profits which can be obtained by consolidating demands, combining delivery routes and collectively serving new customers. First, we try to determine a best-case profit each coalition can obtain. However, if the best-case core is empty, we try to determine a solution which in some sense is close to the best-case core.

2. The JNVRG with Optional Customers

In order to introduce the joint network vehicle routing game with optional customers (JNVRGOC), we first introduce the grouped capacitated profitable tour problem with contract customers (GCPTPCC).

Consider a complete directed graph \( G = (V, A) \). The vertex set \( V \) is defined as \( \{0\} \cup V^c \cup V^o \), where 0 represents the depot, \( V^c \) represents the set of contract customers and \( V^o \) represents the set of optional customers. The set of optional customers \( V^o \) is partitioned in \( m \) non-empty and disjunct sets \( V^o_l \) with \( l \in M = \{1, \ldots, m\} \). A reward \( p^o_l > 0 \) with \( l \in M \) is only obtained if all customers of \( V^o_l \) are visited. In contrast to the optional customers, the contract customers all have to be visited and no reward is given for visiting these customers. With each arc \( (v, w) \in A \), a travel cost \( c_{vw} \geq 0 \) is associated, which we assume to adhere to the triangle inequality. Furthermore, each customer \( v \in V^c \cup V^o \) has a demand \( d_v > 0 \). A limited number of vehicles \( k \) with capacity \( Q \) is available at the depot to satisfy the demand by visiting customers along routes. A route is defined as a simple cycle, starting and ending at the depot, such that the total demand does not exceed the capacity.

Here, we assume that \( d_v \leq Q \) for all \( v \in V^c \cup V^o \).

The GCPTPCC is the problem of constructing routes in such a way that the profit, i.e., the total rewards minus the total costs, is maximised and every contract customer \( v \in V^c \) is visited. We denote the optimal objective value of a GCPTPCC on the graph induced by the vertex sets \( V^o \) and \( V^c \) as GCPTPCC\((V^o, V^c)\). To the best of our knowledge, the GCPTPCC has not been
studied before. However, van Zon, Spliet, and van den Heuvel (2019) study the grouped capacitated profitable tour problem, which can be seen as a special case of the GCPTPCC in which no contract customers are considered.

Next, we define a cooperative game based on the GCPTPCC, which we call the JNVRGOC. A cooperative game is defined as a pair \( \langle N, v \rangle \) where the grand-coalition \( N = \{1, \ldots, n\} \) denotes the set of players and \( v : 2^N \to \mathbb{R} \) denotes the profit of a coalition \( S \subseteq N \). Here, each player \( i \in N \) is assumed to have a finite fleet of \( k_i \) vehicles with capacity \( Q \) available. Hence, a coalition \( S \subseteq N \) has \( \sum_{i \in S} k_i \) vehicles available. Moreover, we also assume each player \( i \in N \) to have a set of contract customers \( V_i^c \) of which the demand has to be fulfilled, with \( V_i^c \cap V_j^c = \emptyset \) for \( i, j \in N \).

The profit of a coalition \( S \subseteq N \) depends on how the sets of optional customers are allocated among the coalition \( S \) and the remaining players \( N \setminus S \). Here, we consider the best-case profit \( v(S) \) that coalition \( S \) can obtain. In the best case, coalition \( S \) can freely select sets of optional customers to serve. We define the best-case profit \( v(S) \) as follows

\[
v(S) = \text{GCPTPCC} \left( V_o \cup \bigcup_{i \in S} V_i^c \right) + \sum_{i \in S} p_i^c,
\]

where we add the stand-alone routing costs \( p_i^c = -\text{GCPTPCC}(\emptyset, V_i^c) \) of the contract customers belonging to the coalition, in order to ensure that the value created by the coalition is properly represented by \( v(S) \). We define the JNVRGOC as a cooperative game \( \langle N, v \rangle \).

3. Solution approach

We aim to find an allocation for the JNVRGOC in the best-case core. However, if the best-case core is empty, we try to find an allocation which in some sense is close to the best-case core. To this end, we consider the following linear programming problem

\[
\begin{align*}
\max \quad & \alpha, \\
n.t. \quad & y(S) \geq \alpha v(S) \quad & \forall S \subseteq N, \\
& y(N) = v(N), \\
& 0 \leq \alpha \leq 1, \\
& y_i \geq 0 \quad & \forall i \in N.
\end{align*}
\]

We try to maximise \( \alpha \) in order to obtain an allocation which is close to the best-case core. Constraints (3) enforce the rationality constraints given \( \alpha \), whereas Constraint (4) is the efficiency constraint. Moreover, Constraint (5) ensures that \( \alpha \) is within \([0, 1]\) and Constraints (6) ensure that each player is allocated a profit. Note that if there exists an optimal solution \( \alpha' \) and \( y' \) for which \( \alpha' = 1 \), it holds that \( y' \) is in the best-case core. We determine an optimal solution to (2)-(6) both by
means of row generation of Constraints (3) and enumeration of these constraints. Both algorithms are implemented by means of a branch-and-price-and-cut algorithm which include state-of-the-art acceleration strategies.

4. Results and conclusion
We have considered preliminary experiments based on a limited subset of Augerat A instances. For these instances, we divided the customers into \( n \) groups of contract customers as well as \( m \) groups of optional customers, with \( n \in \{5, 7\} \) and \( m \in \{3, 5, 7\} \). Moreover, we assigned each player \( i \in \{1, \ldots, n\} \) precisely enough vehicles to cover its own customers, 

\[
k_i = \left\lceil \frac{\sum_{v \in V^c_i} d_v}{Q} \right\rceil.
\]

Our results show that the row generation algorithm is on average 2 times faster than enumeration for instances with 5 players and over 4 times faster for instances with 7 players. Exponentially better gains are to be expected when the number of players increases any further. Moreover, the average value of \( \alpha \) is 0.83, and 11% of the optimal solutions were in the best-case core.

References


