On the Interplay between Self-driving Cars and Public Transportation: A Game-theoretic Perspective

Nicolas Lanzetti  
Automatic Control Laboratory, ETH Zürich, Physikstrasse 3, 8092 Zürich, Switzerland, nlanzetti@ethz.ch

Maximilian Schiffer  
TUM School of Management, Technical University of Munich, Arcisstrae 21, 80333 Munich, Germany, schiffer@tum.de

Michael Ostrovsky, Marco Pavone  
Stanford University, USA, ostrovsky@stanford.edu, pavone@stanford.edu

1. Motivation
All around the world, (mega) cities struggle with overstrained transportation systems whose externalities cause economic and environmental harm such as working hours lost in congestion or health dangers caused by particulate matters, NO$_x$, and stress [Frakt 2019]. Municipalities try to resolve this problem by improving existing transportation systems but face several obstacles, mainly consisting of spatial limitations in urban areas and decade-long lead times of infrastructure projects [Hu 2019]. Accordingly, a better utilization of current urban infrastructure by means of new technologies and mobility concepts is necessary to resolve the root cause of problems in today’s transportation systems.

Under this backdrop, autonomous mobility-on-demand (AMoD) systems are seen as a promising solution. An AMoD system consists of a fleet of robotic self-driving cars, which transport passengers from their origins to their destinations. A single operator controls the whole fleet by dispatching passenger trips to vehicles while simultaneously deciding on their routes. Most experts agree on AMoD systems to call for a paradigm shift in today’s transportation systems, allowing for a more efficient, socially-aware, and sustainable mobility [Fagnant and Kockelman 2015]. However, skeptics question if such a disruptive change in the mobility marketplace may excessively cannibalize other, yet necessary, transportation modes such as public transport.

Contribution: We provide the first game-theoretical framework that focuses on the dynamics among multiple mobility service providers (MSPs) and customers while considering operational constraints within the system. Specifically, our contribution is fourfold. First, we develop a generic mathematical framework that considers the interdependence between a transportation network and its corresponding market place by connecting graph-theoretical network models with a game-theoretical approach. Second, we tailor this framework to the specific case of an AMoD system interacting with public transport. Third, we develop a computationally tractable quadratic program which yields the equilibrium prices for this specific game. Fourth, we provide a real-world case study
for the city of Berlin, Germany and present extensive numerical results and sensitivity analyses on the interaction between an AMoD system and public transport.

2. Problem Setting

We focus on intra-city passenger transportation, where an AMoD fleet substitutes the service of current taxi or ride-hailing fleets. In such a system commercial MSPs and municipalities interact with each other and with customers. To capture the dynamics of such a system and interactions between stakeholders, we model the city’s transportation system on two different levels: a transportation network and its corresponding market place (see Figure 1).

**Transportation market place:** The interaction between the different stakeholders takes place in the system’s *market place*, realized, e.g., via a smartphone app. Here, MSPs and municipalities offer different types of transportation services to customers. Customers have different transportation demands and respond to these offers depending on their individual rationale. These interactions happen on the operational level in a short time horizon. On the strategic level, MSPs interact and therefore influence each other as their business models may interfere; i.e., a customer may substitute the service of one provider with the service of another provider.

**Transportation network:** The realization of a demand and supply match between customers and mobility service providers takes place in the transportation network which consists of the city’s road and public transportation networks. Accordingly, the transportation network imposes boundaries on offers in the transportation market place as it determines the available infrastructure.

3. Approach

We represent a transportation network on a multigraph $G = (V, A, o, d)$ with a vertex set $V$, an arc set $A$, and identifiers $o : A \to V$ and $d : A \to V$ assigning each arc to its source and sink vertex (see Figure 2). Each vertex $v \in V$ denotes a location where a customer can start or end her trip. Each arc $a \in A$ represents a certain transportation mode for a trip, e.g., a self-driving car or a subway line. We define arc subsets $A_j \subset A$, each defining a subgraph $G_j = (V_j, A_j, o_j, d_j)$ with $V_j := \bigcup_{a \in A_j} \{ o(a), d(a) \}$, $o_j := o|_{A_j}$, and $d_j := d|_{A_j}$. Each subgraph $G_j$ denotes a homogeneous mode...
of transportation, controlled by an MSP (e.g., through price setting), henceforth referred to as its operator. The subgraph $G_0 = (V_0, A_0, o_0, d_0)$ denotes a part of the overall network where customers can move free of charge, e.g., by walking along the streets.

Operators and customers interact sequentially: First, the customers set requests in $G_0$. Second, the MSPs decide on prices for their offered services on their subgraphs $G_j$. Third, the customers choose a transportation service which then results in transport flows on $G$.

Customers: We identify customer demands by triples $q_i = (o_i, d_i, \alpha_i)$, where $o_i$ is the origin vertex, $d_i$ is the destination vertex, and $\alpha_i$ the demand rate (per unit time). Customers may travel free of charge by using arcs in the subgraph $G_0$ (i.e., they walk) or may request an MSP’s service. We model their decision through a reaction curve $\phi_i(p)$, assigning a rate to each path $p$ connecting the customers’ origin and destination. To each reaction curve we associate a cost, reflecting the customers’ individual benefit (e.g., a combination of a fare and a monetary value of time).

Operators: We model the operators’ decision through a pricing strategy $\xi_j$, mapping each origin-destination pair $(o, d) \in V_j \times V_j$ to a non-negative price. Each operator selects the pricing strategy in order to maximize her profit.

We characterize the equilibria of a game with $M$ demands and $N_o$ operators as follows: We say that $((\phi^*_i)_{i=1}^{M}, (\xi^*_j)_{j=1}^{N_o})$ is an equilibrium if each demand $q_i$ will incur in a larger cost by unilaterally deviating to a reaction curve $\phi_i \neq \phi^*_i$, and each operator’s profit will decrease by unilaterally deviating to a pricing strategy $\xi_j \neq \xi^*_j$.

4. Experimental Studies

In this work, we specify this framework for the interplay among an AMoD operator, public transport, and customers. Herein, the AMoD operator offers mobility services on the road network and the municipality offers public transport on a parallel infrastructure, e.g., a subway or a tram network.
We base our studies on a real-world case for the central urban area of Berlin, Germany and use demand data from an existing case study [Horni, Nagel, and Axhausen 2016; Ziemke and Nagel 2017], consisting of 129,560 travel requests. Beyond studying a basic scenario, we investigate potential AMoD operator strategies to influence the equilibrium of the system and potential strategies a municipality may use to counteract the AMoD operator.

Our experiments reveal that the modal share at equilibrium splits nearly equally between AMoD system and public transport. If the AMoD fleet size is not limited, the share of customers taking the AMoD system can increase up to 76%. Local cannibalization can deviate from the macroscopic effects. Indeed, nearly 22.5% of the trips are entirely served by the AMoD system, 38.9% are served solely by public transport, and only 34.7% show a modal split that correlates with the macroscopic findings. Our studies showed that a (non-autonomous) mobility-on-demand system generates less than 10% of the profit and serves less than 25% of the customers compared to its autonomous counterpart. We show that a free public transportation system reduces the AMoD modal share and profit by about 20% and 70%, respectively. A service tax imposed on the revenue of each AMoD trip lower than 60% diminishes the profit generated by the AMoD operator, yet it does not shift the modal split.

5. Conclusions
We studied the interplay between AMoD fleets and public transportation. To this end, we developed a general methodological framework to model interactions among MSPs and between MSPs and customers. Herein, we combined graph-theoretical network flow models with a game-theoretical approach to capture both the interactions between MSPs and customers on the transportation market place and the constraints that result from the transportation network. We specified this framework for the interplay among an AMoD fleet operator, a municipality, and customers. We developed a computationally tractable quadratic program to find the equilibrium of the resulting game and applied our methodology to a real-world case study for the city of Berlin, Germany to derive managerial insights for academics and practitioners.

References