Pricing of Shared Autonomous Vehicles Systems with Elastic Demand

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1. Introduction and Background

Car sharing is a broad idea of sharing a fleet of vehicles that has attracted attentions of researchers for its potential to revolutionize the current transportation systems (Jorge et al. 2012). A successful car sharing systems have the ability to provide systematic benefits such as increasing vehicle utilization rate, reducing cost for commuting transportation and increasing users’ mobility (Litman 2000). With rapid advance of the Autonomous Vehicles (AVs) technology, the Shared Autonomous Vehicles (SAVs) Systems, likely with operator owned fleet and centralized control systems, have been accepted. Many aspects of today’s transportation systems will be transformed with the introduction of SAVs systems (Daniel J. Fagnant and Bansal 2015). With the robotic fleet replacing human drivers, we expect that the cost of using SAVs services to be driven down to the point that makes it possible for mass adoption (Keeling 2018).

Providing service to all demand is not an efficient solution even with AVs and central control as travel demand is highly unbalanced temporally and spatially. This is evident in current operations of ride sourcing systems such as Uber and Lyft surge pricing, uneven distribution of demand coverage and unexpected long waiting time that may cause reneging of potential customers (Bocken et al. 2018). As SAVs grow their shares of covering travel demand along with Private Autonomous Vehicles (PAVs), SAVs can be operated to proactively manage travel demands. In particular, we assume that travelers intending to use SAVs will resort to PAVs if SAVs service is based on generalized cost composed of waiting time and pricing. That is, SAVs systems can be operated to

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balance between demand coverage and system objective (revenue maximizing) (Cherubini et al. 2015). To our best knowledge, although some papers consider the pricing in car-sharing as shown in Table 1, few literature have considered reneging dynamics within an optimization. Then our research is focus on developing an integrated SAVs systems pricing and operations (e.g., fleet sizing, fleet operation, and vehicle redistribution) framework that endogenously handles customers that may reneging based on prices and waiting time.

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Table 1 Previous Studies about Car-Sharing Pricing

2. Methodology

Our proposed SAVs systems pricing problems are built upon operational and pricing models where the predetermined maximum service waiting time $M$ constraints the total demand covered by SAVs. In Section 2.1, demand is assumed to be fixed (i.e., not elastic) and SAVs serve the demand in $M$ time periods. In Section 2.2, customers may leave SAVs systems to choose PAVs instead causing the elastic demand but does not wait longer than $M$ time periods.

2.1. SAVs Operational Model: Baseline Case

A Time-Space-Network Shared Autonomous Vehicles Systems (TSN-SAVs) operations model is developed for SAVs systems where pricing is given as an input. This formulation will serve as a core model where pricing models are built upon. In order to serve as a building block for the proposed pricing model, this operational model should not only includes capabilities of determining 1) fleet sizing, $V_i(t)$, 2) flow of vehicles including those in service, $X_{i,j}(t)$, as well as redistribution and pre-positioning, but also 3) travelers’ demand for SAVs, $D_{i,j}^{SAVs}(t)$ that is a function of prices as well as the waiting time compared between SAVs and PAVs.

\[
\begin{align*}
\text{(TSN-SAVs)} & \quad \min \sum_{(i,j) \in A} \sum_{t \in T} c_{i,j}(t)X_{i,j}(t) + \sum_{(i,j) \in A} \sum_{t \in T} p_{i,j}(t)U_{i,j}(t) + \sum_{(i,j) \in A} \sum_{t \in T} h_{i}(t)V_{i}(t) \\
\text{subject to:} & \quad U_{i,j}(t) \geq U_{i,j}(t-1) + D_{i,j}^{SAVs}(t) - X_{i,j}(t) \quad (i,j) \in A, t \in T
\end{align*}
\]
\[
\min \left( M \cdot |T| - M \right) \sum_{m=0}^{M} X_{i,j}(t + m) \geq U_{i,j}(t - 1) + D_{i,j}^{SAVs}(t) \quad (i,j) \in \mathcal{A}, t \in \mathcal{T}
\]

\[
V_i(t) = V_i(t-1) + \sum_{j \in \mathcal{N}(i)} \sum_{\tau < t} \alpha_{j,i}(\tau, t)X_{j,i}(\tau) - \sum_{j \in \mathcal{N}(i)} X_{i,j}(t) \quad i \in \mathcal{N}, t \in \mathcal{T}
\]

\[
\sum_{i \in \mathcal{N}} V_i(0) = \sum_{i \in \mathcal{N}} V_i(|T|)
\]

\[
X_{i,j}(t), U_{i,j}(t), V_i(t) \geq 0 \quad (i,j) \in \mathcal{A}, t \in \mathcal{T}
\]

The operational objective function (1) is to minimize the total costs including the cost of vehicle operation \(c_{i,j}(t)\), penalty of delaying serving the SAVs demand, \(p_{i,j}(t)\), and vehicle inventory cost, \(h_i(t)\). Constraint (2) states that the unmet demand for the current time period, \(U_{i,j}(t)\) is at least the summation of the cumulative back ordered unmet demand from the previous time period and demand for the current time period minus the flow for the current time period. Constraint (3) guarantees that the demand is served within \(M\) time periods which is the upper bound of service waiting time. Constraint (4) represents flow conservation and Constraint (5) is a fleet size constraint which guarantees the same fleet size throughout the time.

### 2.2. SAVs Operational Model: Elastic Demand

**Demand Composition:** In designing price settings of SAVs, the travel demand needs to be further specified. The total demand from origin \(i\) to destination \(j\) at time \(t\) is given by \(d_{i,j}(t)\) composed of using SAVs \(D_{i,j}^{SAVs}(t)\) and PAVs \((d_{i,j}(t) - D_{i,j}^{SAVs}(t))\) as seen in figure (1).

- **SAVs:** \(D_{i,j}^{SAVs}(t)\)
- **PAVs:** \((d_{i,j}(t) - D_{i,j}^{SAVs}(t))\)
- **All demand:** \(d_{i,j}(t)\)

**Waiting Time:** Based on queuing theory little’s theorem: average waiting time \(W_{i,j}(t)\) can be expressed by SAVs unmet demand in the queue \(U_{i,j}(t)\) and demand served by SAVs \(D_{i,j}^{SAVs}(t)\) where \(L_{i,j}(t)\), \(\lambda\) and \(|T|\) represent customers waiting in the queue, effective arrival rate and set of time period that \(W_{i,j}(t) = \frac{L_{i,j}(t)}{\lambda} = \frac{U_{i,j}(t)}{D_{i,j}^{SAVs}(t)} = \frac{U_{i,j}(t)|T|}{D_{i,j}^{SAVs}(t)}\).

**Generalized Cost:** Travelers are assumed to evaluate travel mode based on the generalized cost. Generalized cost is defined as a linear combination of waiting time and cost. \(C_{SAVs} = \beta_{SAVs} + w_p P_{i,j}(t) + w_w W_{i,j}(t)\) and \(C_{PAVs} = \beta_{PAVs} + w_p P_{i,j}(t)\) where parameters \(w_p, w_w\) are weights between cost and waiting time (positive values) and \(\beta_{SAVs}, \beta_{PAVs}\) are mode specific preferences for SAVs and PAVs respectively. It is noted that \(c_{i,j}(t)\) is the operating cost of PAVs and the cost of SAVs,
with respect to $U$ and price $P$. SAVs service to reduce the cost of possible empty relocations. In this case, the centralized control systems set lower pricing to attract more potential customers that may choose SAVs service to reduce the cost of possible empty relocations.

**Utility Function:** Utility functions for SAVs and PAVs are $U_{SAV_s}$, $U_{PAV_s}$ respectively. These two are designed to be measured by service $q$ (constant) over generalized cost, $U_{SAV_s} = \frac{q}{C_{SAV_s}}$, $U_{PAV_s} = \frac{q}{C_{PAV_s}}$. The reason for designing inverse of generalized cost as utility function is to express demand cover by SAVs, $D_{i,j}^{SAV_s}(t)$ to use unmet demand $U_{i,j}(t)$ and price $P_{i,j}(t)$ to express demand cover by SAVs, $D_{i,j}^{SAV_s}(t)$, in equation (7).

$$D_{i,j}^{SAV_s}(t) = \frac{\beta_{PAV_s} d_{i,j}(t) + c_{i,j}(t) d_{i,j}(t) w_p - w_w |T| U_{i,j}(t)}{\beta_{SAV_s} + \beta_{PAV_s} + w_p (c_{i,j}(t) + P_{i,j}(t))}$$ (7)

It is noted that $D_{i,j}^{SAV_s}(t)$ in operational model from equations (1) through (6) is nonlinear with respect to $U_{i,j}(t), P_{i,j}(t)$. If price is fixed, this becomes a linear expression. We propose a bi-level structure approach that keeps the operational model linear.

### 2.3. SAVs Pricing Model

A bi-level programming approach is proposed in order to keep the formulation linear whose structure between the operator and customers is widely used in pricing studies for transportation services (Haider et al. 2018, Yang and Zhang 2002, Farahani et al. 2013).

$$(\text{Pricing}) \quad \max_P \sum_{t \in T} \sum_{(i,j) \in A} P_{i,j}(t) \cdot D_{i,j}^{SAV_s}(t)$$ (8)

subject to: $P_{i,j}(t) \geq 0 \quad (i,j) \in A, \quad t \in T$ (9)
\[0 \leq D_{i,j}^{SAVs}(t) \leq d_{i,j}(t) \quad (i,j) \in A, \quad t \in T \quad (10)\]

where \(D_{i,j}^{SAVs}(t) = \arg \min_{D,X,U,V} \{ \text{TSN-SAveS: P} \} \quad (11)\)

Notice that upper level is unconstrained and in operational level (TSN-SAveS) \(D_{i,j}^{SAVs}(t)\) is a Linear Programming problem. In the pricing model the objective is to maximize the total revenue by setting proper prices, \(P_{i,j}(t)\) given the fixed number of SAVs demand, \(D_{i,j}^{SAVs}(t)\). The SAVs coverage demand is determined in operational model shown in equation (11), where the prices are given as parameters, essentially through solving TSN-SAveS in equations from (1) through (6).

The proposed formulation as a bi-level is still hard to solve. KKT condition does not work because of \(D_{i,j}^{SAVs}(t)\) in equation (7) is nonlinear as pricing, \(P_{i,j}(t)\), and SAVs unmet demand, \(U_{i,j}(t)\) are decision variables. Therefore we employ an iterative methods where expression (10) is modified based on improving direction. This improving direction algorithm works as initially finding feasible solution and searching improving directions until no more improving directions found. Each improving direction corresponds to a fresh run of operational model problem and an incumbent solution is introduced. In particular we use the dual values of operational model to determine if increasing the coverage of demand is preferred by modifying the constraint (10).

3. Case Study

In this research task, we apply the models to travel forecasting data of New York City (NYC) by New York Metropolitan Transportation Council (NYMTC). We consider 20 zones can be roughly defined for NYC and the demand are also merged accordingly. The demand in NYC forecasting data is defined based on 4 time blocks: morning (AM), mid day (MD), afternoon (PM) and night time (NT). To feed SAVs systems, we need more granular data. Specifically, we define 40 time periods for the time from 6am to 9pm and each time period is 22.5 minutes even.

To make the numerical experiment more realistic, we subdivide the demand from the 4 time blocks in NYC forecasting data to obtain the demand for each of the 40 time periods based on the National Household Travel Survey (NHTS) trip distribution, assuming that the NHTS distribution represents the trip distribution for New York City.

The goal is to derive system insights of how SAVs price setting effects elastic demand considering customer reneging dynamics, but also to maximize SAVs total revenue and better satisfying customer demand. Some of the related questions we plan to answer are as follows:

- What strategy of dynamic SAVs price settings are encouraged to maximize SAVs revenue as well as the overall satisfying customer demand?
- Should there be regulations/incentives on SAVs prices to reduce inefficiencies of SAVs as well as the overall transportation system?
- How to reach dynamic equilibrium between SAVs price settings and customer demand satisfaction level to make SAVs balanced?
References


