1. Introduction

The challenges of last mile delivery differ based on customer geography. In rural areas, customers are further apart resulting in longer routes for the delivery person (Dolan 2018). In urban areas, customers are in close proximity, but the delivery person must deal with traffic congestion and the need to find parking. According to a report by INRIX Research, the average search for parking is 9 minutes in these urban areas (Cookson and Pishue 2017). To mitigate the challenges of last mile delivery, we consider the use of an autonomous vehicle that can remain in continuous use while the delivery person is making on foot deliveries. In this presentation, we examine the value of autonomous vehicles across the rural-urban continuum.

The Capacitated Autonomous Vehicle Assisted Delivery Problem (CAVADP) is the problem of serving \( n \) customers using an autonomous vehicle assisted by a delivery person. The delivery person can carry up to \( q \) packages and must coordinate with the vehicle to replenish when necessary. The vehicle can travel between customers with or without the delivery person on board. The autonomous nature of the vehicle allows for the delivery person to load at a given customer, serve a set of customers by foot, and be picked up by the vehicle at an alternate location. This flexibility eliminates the need for the driver to be on board to find parking.

To analyze the results of the CAVADP, we introduce the Capacitated Delivery Problem with Parking (CDPP) as a benchmark. The CDPP also serves \( n \) customers but the vehicle is required to be accompanied by the delivery person while in use. In order for the delivery person to serve a set of customers, the vehicle must be parked. After parking and delivering to a set of customers, the
delivery person can return to the same parked location to reload packages and deliver to another set of customers or find a new parking spot from which to serve the next set of customers. However, moving to a new parking spot comes at the cost of finding parking again. The goal of the CAVADP and CDPP is to minimize the completion time of the delivery tour, i.e. the time that the delivery person is with the vehicle as well as the time spent walking to serve n customers.

Both problems are modeled as integer programs. In previous work, we characterize the optimal solution to the CAVADP on a solid rectangular grid of customers based on the number of customers n, driving speed of the vehicle d, walking speed of the delivery person w, and the fixed time for loading packages f (Reed, Campbell, and Thomas 2019). Computational experiments using these results demonstrate that the CAVADP reduces the completion time of the CDPP on the grid by 30-77%, with higher values achieved for longer parking times, smaller capacities, and lower fixed time for loading packages.

This talk considers when customers are no longer restricted to a grid. The United States Department of Agriculture classifies counties on a rural-urban continuum based on population size, degree of urbanization, and adjacency to metro areas (Agr 2019). Once we understand how to solve a more general instance, we can quantify potential savings from this use of autonomous vehicles along this rural-urban continuum.

The contributions of this work can be summarized as follows:

• Valid inequalities for the CAVADP on a general geography
• Analysis of the value of autonomous vehicles on the rural-urban continuum.

2. Solution Methodology for CAVADP on General Geography

On the route of the delivery person in the CAVADP, he/she is either on board the vehicle or walking from customer i to customer k. We construct the driving graph $G_d = (V_d, E_d)$ where $V_d = \{0, 1, ..., n\}$ to represent the option for the delivery person to be on board the vehicle. Note that the delivery person can be on board the vehicle between all locations (including the depot). Therefore, $(i, k) \in E_d$ for all $i, k \in V_d$ and $G_d$ is a complete graph.

Observe that if it takes less time to drive from customers i to k and load package(s) at k than to walk from i to k, the delivery person would never walk between i and k. This observation can be written as the following inequality:

$$\frac{1}{d} D(i, k) + f < \frac{1}{w} D(i, k).$$

(1)

From this observation, we construct the walking graph $G_w = (V_w, E_w)$ where $V_w = \{1, ..., n\}$. For $(i, k) \in V_w$, there exists edge $(i, k) \in E_w$ iff $D(i, k) < \frac{d}{d-\frac{f}{w}}$. This edge represents the potential to
walk from customer \( i \) to \( k \). Otherwise, the analysis of Inequality (1) shows that it is always better to drive between \( i \) and \( k \), meaning the delivery person would never walk from \( i \) to \( k \).

Figure 1 gives walking graphs for three different counties on the rural-urban continuum defined by the Department of Agriculture (Agr 2019). Figure 1a gives a complete graph meaning the delivery person is willing to walk between all customers. Figure 1b shows urban instances can result in multiple connected components. Finally, Figure 1c gives a rural instance where most points are isolated. In general, more urban instances have walking graphs with nodes of higher degree.

Properties of the walking graph reveal structure in the optimal solution. For example, if there is an isolated point \( i \) in the walking graph, all customers are “far enough” away that the delivery person will never walk to customer \( i \) from any other customer. Therefore, customer \( i \) will be served individually. In fact, the individual service of customer \( i \) requires the delivery person to be on board the vehicle to and from \( i \). This idea extends to servicing more than one customer on foot between loading points. If there does not exist a path in the walking graph between these customers, then this set of customers will not be served in the optimal solution. At some point during service, the delivery person would rather drive and load packages for the remaining customers than walk. These observations reduce the size of the model and determine structure in the optimal solution.

Figure 2 shows the reduction in computational time for \( n = 100 \) customers. Including information from the walking graph reduces the computational time up to 99%. We also establish valid inequalities based on the delivery person’s decision to either walk or drive. The valid inequalities reduce the integrality gap by up to 73%. In combination with the walking graph, the valid inequalities additionally reduce the computational time up to 96%.

3. Conclusions

The ability to solve the general instance of the CAVADP allows for analysis of the value of autonomous vehicles on the rural-urban continuum. Figure 3 shows average savings when considering variations in driving speed and parking times. For \( n = 50 \) customers, we show the CAVADP reduces the completion time of the CDPP on average 21-41% with higher savings for longer parking times. To solidify our preliminary findings, future work includes solving the CDPP on larger
instances to understand the effects of the parameters on the parking case. Advances with the CAVADP and CDPP will be valuable in understanding how to mitigate challenges in last mile delivery and best use this new technology.

References


Reed S, Campbell A, Thomas B, 2019 The Value of Autonomous Vehicles for Last Mile Deliveries in Urban Environments.